MATHEMATICS

Grade 7

Part - I

Educational Publications Department

To obtain electronic textbooks, visit
www.edupub.gov.lk

First Print	-	2015
Second Print	-	2016
Third Print	-	2017
Fourth Print	-	2018
Fifth Print	-	2019

ISBN 978-955-25-0102-9

Published by Educational Publications Department
Printed by Fast Printery (Pvt) Ltd.
No. 165, Devanampiyatissa Mawatha, Colombo 10

The National Anthem of Sri Lanka

Sri Lanka Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Sundara siri barinee, surendi athi sobamana Lanka
Dhanya dhanaya neka mal palaturu piri jaya bhoomiya ramya
Apa hata sepa siri setha sadana jeewanaye matha
Piliganu mena apa bhakthi pooja Namo Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha
Oba we apa vidya
Obamaya apa sathya
Oba we apa shakthi
Apa hada thula bhakthi
Oba apa aloke
Apage anuprane
Oba apa jeevana we
Apa mukthiya oba we
Nava jeevana demine, nithina apa pubudukaran matha
Gnana veerya vadawamina regena yanu mana jaya bhoomi kara
Eka mavakage daru kela bevina
Yamu yamu vee nopama
Prema vada sema bheda durerada
Namo, Namo Matha
Apa Sri Lanka Namo Namo Namo Namo Matha

අපි වෙමු එක මවකගෙ දරුවෝ
එක නිවසෙහි වෙසෙනා
එක පාටැති එක රුධිරය වේ
අප කය තුළ දුවනා

එබැවිනි අපි වෙමු සොයුරු සොයුරියෝ
එක ලෙස එහි වැඩෙනා
ජීවත් වන අප මෙම නිවසේ
සොඳින සිටිය යුතු වේ

සැමට ම මෙත් කරුණා ගුණෙනී
වෙළී සමගි දමිනී
රන් මිණි මුතු නො ව එය ම ය සැපතා
කිසි කල නොම දිරනා

ආනන්ද සමරකෝන්

ஒரு தாய் மக்கள் நாமாவோம்
ஒன்றே நாம் வாழும் இல்லம்
நன்றே உடலில் ஓடும்
ஒன்றே நம் குருதி நிறம்

அதனால் சகோதரர் நாமாவோம்
ஒன்றாய் வாழும் வளரும் நாம்
நன்றாய் இவ் இல்லினிலே
நலமே வாழ்தல் வேண்டுமன்றோ

யாவரும் அன்பு கருணையுடன்
ஒற்றுமை சிறக்க வாழ்ந்திடுதல்
பொன்னும் மணியும் முத்துமல்ல - அதுவே
யான்று மழியாச் செல்வமன்றோ.

ஆனந்த சமரக்கோன்
கவிதையின் பெயர்ப்பு.

Being innovative, changing with right knowledge
Be a light to the country as well as to the world.

Message from the Hon. Minister of Education

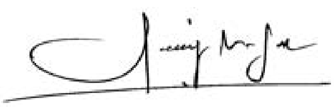
The past two decades have been significant in the world history due to changes that took place in technology. The present students face a lot of new challenges along with the rapid development of Information Technology, communication and other related fields. The manner of career opportunities are liable to change specifically in the near future. In such an environment, with a new technological and intellectual society, thousands of innovative career opportunities would be created. To win those challenges, it is the responsibility of Sri Lankan Government and myself, as the Minister of Education, to empower you all.

This book is a product of free education. Your aim must be to use this book properly and acquire the necessary knowledge out of it. The government in turn is able to provide free textbooks to you, as a result of the commitment and labour of your parents and elders.

Since we have understood that the education is crucial in deciding the future of a country, the government has taken steps to change curriculum to suit the rapid changes of the technological world. Hence, you have to dedicate yourselves to become productive citizens. I believe that the knowledge this book provides will suffice your aim.

It is your duty to give a proper value to the money spent by the government on your education. Also you should understand that education determines your future. Make sure that you reach the optimum social stratum through education.

I congratulate you to enjoy the benefits of free education and bloom as an honoured citizen who takes the name of Sri Lanka to the world.



Akila Viraj Kariyawasam
Minister of Education

Foreword

The educational objectives of the contemporary world are becoming more complex along with the economic, social, cultural and technological development. The learning and teaching process too is changing in relation to human experiences, technological differences, research and new indices. Therefore, it is required to produce the textbook by including subject related information according to the objectives in the syllabus in order to maintain the teaching process by organizing learning experiences that suit to the learner needs. The textbook is not merely a learning tool for the learner. It is a blessing that contributes to obtain a higher education along with a development of conduct and attitudes, to develop values and to obtain learning experiences.

The government in its realization of the concept of free education has offered you all the textbooks from grades 1-11. I would like to remind you that you should make the maximum use of these textbooks and protect them well. I sincerely hope that this textbook would assist you to obtain the expertise to become a virtuous citizen with a complete personality who would be a valuable asset to the country.

I would like to bestow my sincere thanks on the members of the editorial and writer boards as well as on the staff of the Educational Publications Department who have strived to offer this textbook to you.

W. M. Jayantha Wickramanayaka,
Commissioner General of Educational Publications,
Educational Publications Department,
Isurupaya,
Battaramulla.
2019.04.10

Monitoring and Supervision

W.M. Jayantha Wickramanayaka - Commissioner General of Educational Publications
Educational Publications Department

Direction

W. A. Nirmala Piyaseeli - Commissioner of Educational Publications (Development)
Educational Publications Department

Co-ordination

Mrs. H. Chandima Kumari de Zoysa - Assistant Commissioner
Educational Publications Department

Mrs. T. D. C. Kalhari Gunasekara - Assistant Commissioner
Educational Publications Department

Panel of Editors

Dr. Romaine Jayewardene - Senior Lecturer
Department of Mathematics, Faculty of Science
University of Colombo

Dr. R. T. Samarathunga - Senior Lecturer
Department of Mathematics, Faculty of Science
University of Colombo

Dr. Naleen Ganegoda - Senior Lecturer
Faculty of Applied Science
University of Sri Jayewardenepura

B. D. Chitthananda Biyanwila - Director of Education (Mathematics)
Ministry of Education

M. N. P. Pieris - Lecturer
National Institute of Education

S. Rajendran - Lecturer
National Institute of Education

H. Chandima Kumari De Zoysa - Assistant Commissioner
Educational Publications Department

T. D. C. Kalhari Gunasekara	- Assistant Commissioner Educational Publications Department

Panel of Writers

B. L. Samarasekara	- Lecturer Department of Mathematics, Faculty of Science University of Colombo
Anuradha Mahasinghe	- Lecturer Department of Mathematics University of Colombo
Dr. Jayampathi Rathnayake	- Lecturer Department of Mathematics University of Colombo
K. U. S. Somarathna	- Lecturer Faculty of Engineering, University of Moratuwa
Anura D. Weerasinghe	- I.S.A (Piriwena) Matara District
B. M. Bisomenike	- I.S.A. Divisional Education Office, Wariyapola
B. L. Mithrapala	- Assistant Director of Education Zonal Education Office, Hakmana
Ajith Ranasinghe	- I.S.A. Zonal Education Office, Homagama
Mervyn Ruberu Gunasekara	- Principal (Retired)
D. Liston Silva	- Principal (Retired)
M. S. Rafeethu	- I.S.A (Retired)
U. Vivekananthan	- Principal, Sinhala Vidyalaya, Dikoya

R. S. E. Pushparajan - Assistant Director of Education (Retired)

H. Chandima Kumari De Zoysa - Assistant Commissioner
Educational Publications Department

Proof Reading

Asirini De Mel - Ladies' College, Colombo 7
S. D. C. Nilushi Siripala - Musaeus College, Colombo 7

Computer Technology

B. A. Chalani Yuranga - Educational Publications Department
P. D. Piyumi Hansika - Educational Publications Department
W. A. Poorna Jayamini - Educational Publications Department

Cover Design
R. M. Rajitha Sampath - Educational Publications Department

Message of the Boards of Writers and Editors

This textbook has been compiled in accordance with the new syllabus which is to be implemented from 2016 for the use of grade seven students.

We made an effort to develop the attitude "We can master the subject of Mathematics well" in students.

In compiling this textbook, the necessity of developing the basic foundation of studying mathematical concepts in a formal manner was specially considered. This textbook is not just a learning tool which targets the tests held at school. It was compiled granting due consideration to it as a medium that develops logical thinking, correct vision and creativity in the child.

Furthermore, most of the activities, examples and exercises that are incorporated here are related to day to day experiences in order to establish mathematical concepts in the child. This will convince the child about the importance of mathematics in his or her daily life. Teachers who guide the children to utilize this textbook can prepare learning tools that suit the learning style and the level of the child based on the information provided here.

Learning outcomes are presented at the beginning of each lesson. A summary is provided at the end of each lesson to enable the child to revise the important facts relevant to the lesson. Furthermore, at the end of the set of lessons related to each term, a revision exercise has been provided to revise the tasks completed during that term.

Every child does not have the same capability in understanding mathematical concepts. Thus, it is necessary to direct the child from the known to the unknown according to his / her capabilities. We strongly believe that it can be carried out precisely by a professional teacher.

In the learning process, the child should be given ample time to think and practice problems on his or her own. Furthermore, opportunities should be given to practice mathematics without restricting the child to just the theoretical knowledge provided by mathematics.

Our firm wish is that our children act as intelligent citizens who think logically by studying mathematics with dedication.

Boards of Writers and Editors

Contents

1 Symmetry

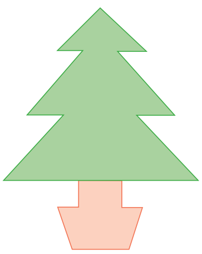
By studying this lesson you will be able to

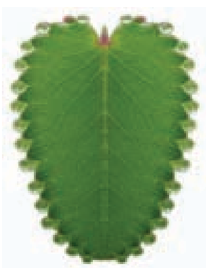
- identify plane figures with bilateral symmetry,
- draw the axes of symmetry of a bilaterally symmetric figure, and
- create bilaterally symmetric figures on square ruled paper.

1.1 Bilateral Symmetry

A figure of a blue quadrilateral shaped card is given here. By folding this figure along the dotted line, we obtain two parts that coincide on each other.

A few figures having two parts which coincide with each other when folded along a certain line are shown below.





Many of the objects in the environment have the property that they can be divided into two equal parts. Most creations too have this property which helps preserve their beauty. Let us learn more about plane figures and laminas with a plane figure as the boundary, that have this property.

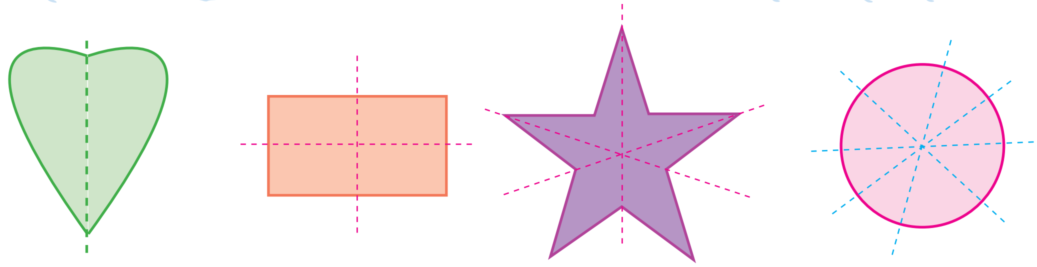


Figure 1 Figure 2 Figure 3 Figure 4

In figure 1, there is only one line that divides the figure into two equal parts which coincide. However, each of the figures 2, 3 and 4, has more than one line that divides the figure into two parts which coincide.

Activity 1

Step 1 - Trace this figure onto a paper and cut along the border of the figure.

Step 2 - Fold the figure so that you get two equal parts which coincide as shown in figure 2.

Step 3 - Draw a dotted line along the fold. Now paste the figure in your exercise book.

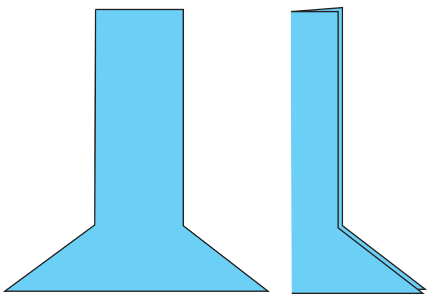
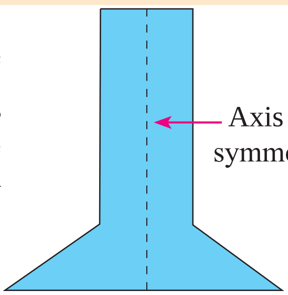


Figure 1 Figure 2

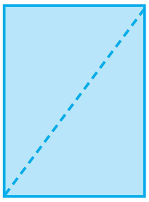
If a plane figure can be folded along a straight line so that you get two parts which coincide, then that plane figure is defined as **a bilaterally symmetric plane figure**. The line of folding is defined as **an axis of symmetry** of the figure.

In the above activity you must have drawn the dotted line shown in the figure as the line along the fold. This line is an "axis of symmetry of the figure". This bilaterally symmetric figure has only one axis of symmetry.



In a bilaterally symmetric figure, the two parts on either side of an axis of symmetry are of the same shape and of the same area.

The figure depicts a rectangle with a dotted line drawn across it. This line divides the rectangle into two equal parts. However if we fold the rectangle along the dotted line, the two parts will not coincide. Therefore this line is not an axis of symmetry of the figure.

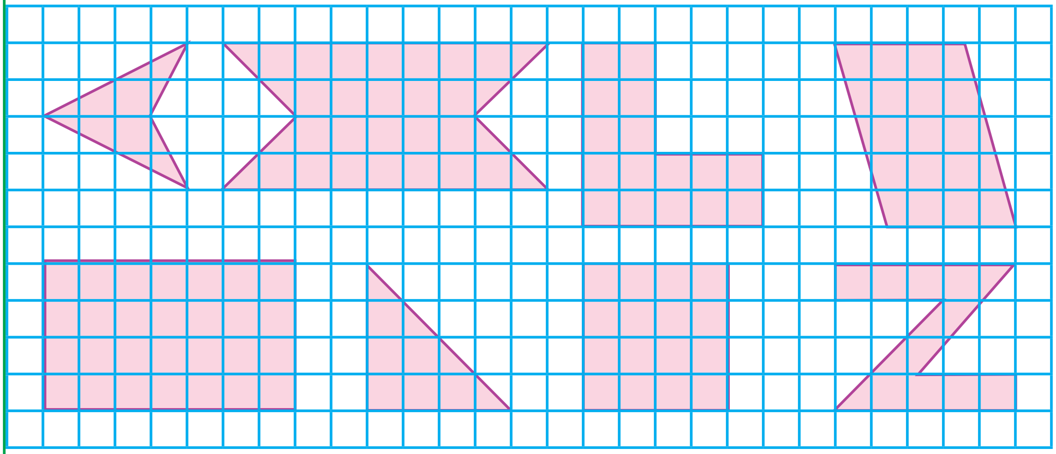


A line through a plane figure which divides it into two parts of the same shape and of the same area which do not coincide with each other is not an axis of symmetry of the figure.

1.2 Drawing axes of symmetry

Activity 2

Step1 - Copy the figures given below on a piece of paper and cut out each lamina.



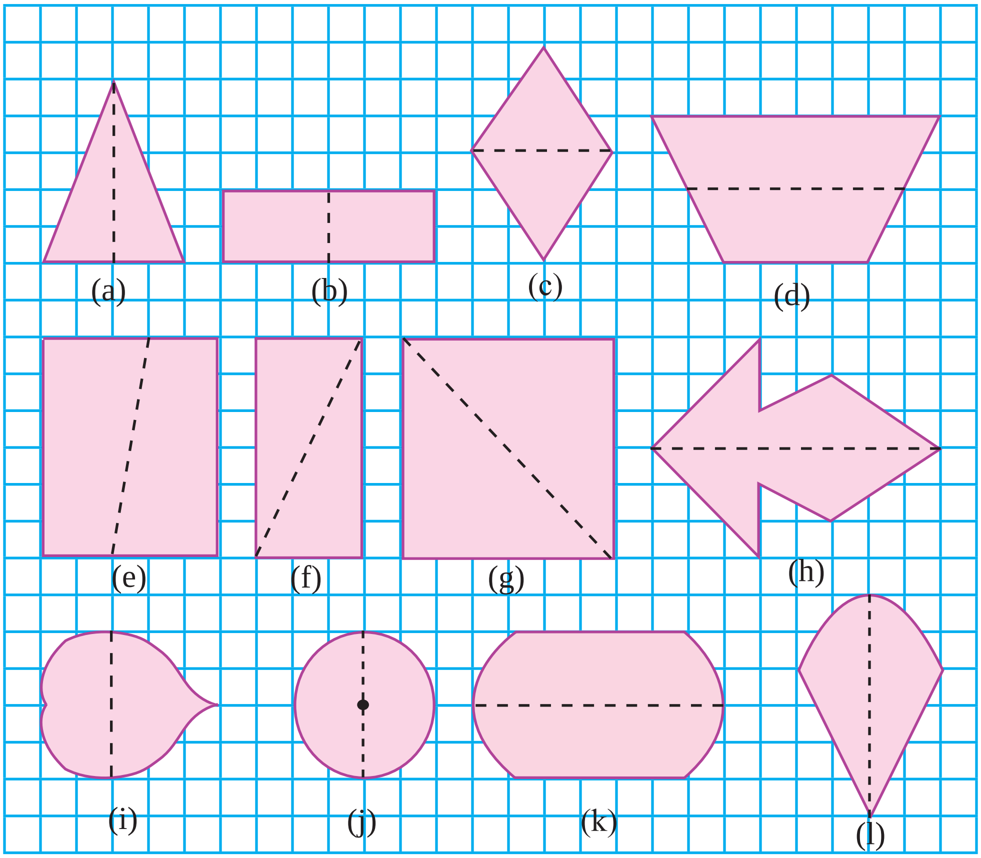
Step 2- Find the bilaterally symmetric figures from the figures that were cut out.

Step 3 - Draw all the axes of symmetry of the figures with bilateral symmetry.

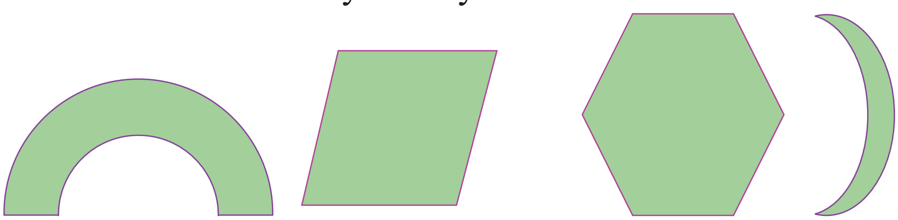
Step 4 - Paste all the figures having axes of symmetry in your exercise book. Near each figure, write its number of axes of symmetry.

Exercise 1.1

(1) From the following, choose the bilaterally symmetric figures with a correctly drawn axis of symmetry and write down the corresponding letters.



(2) (i) Cut out laminas of the following shapes using a tissue paper. Draw all the axes of symmetry of each of them.

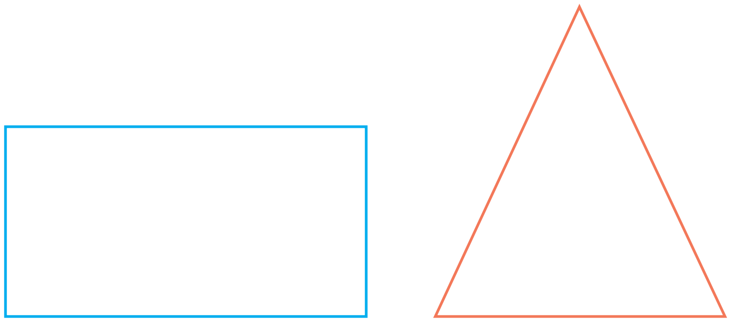


(ii) Paste all the figures having axes of symmetry in your exercise book.

(3) (i) Cut laminas of the following shapes using paper. Draw all the axes of symmetry of each of them.

A - Rectangular shape

B - Triangular shape with two sides of equal length



A *B*

(ii) Write the number of axes of symmetry in each of the above figures.

(iii) Create another symmetric figure by joining two figures of the shapes given in *A* and *B* above and paste it in your exercise book.

(4) Write the statements below in your exercise book. Mark a ✓ in front of the correct sentences and a × in front of the incorrect ones.

(i) In a bilaterally symmetric figure, the two parts on either side of an axis of symmetry are equal in shape and in area.

(ii) There are bilaterally symmetric figures having more than one axis of symmetry.

(iii) The number of axes of symmetry in a circular lamina is greater than the number of axes of symmetry in a square.

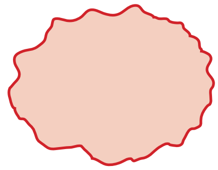
(iv) The maximum number of axes of symmetry in a bilaterally symmetric figure is one.

(v) If a bilaterally symmetric figure which has at least two axes of symmetry is cut along one axis and divided into two equal parts, then each of these parts too will be bilaterally symmetric.

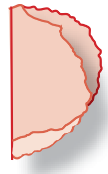
1.3 Creating plane figures having bilateral symmetry

Activity 3

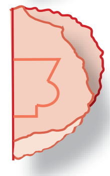
Step 1 - Get a piece of paper of any shape and a pair of scissors.



Step 2 - Fold the paper into two.

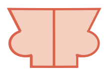


Step 3 - Draw any figure of your choice such that it contains a part of the line of folding and is limited to the area where the two portions overlap (see diagram).



Step 4 - Cut out the figure you drew.

Step 5 - Unfold the figure.



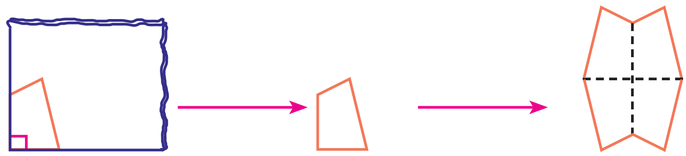
At the end of the above activity you obtain a bilaterally symmetric figure. Its axis of symmetry is the initial line along which you folded the paper.

Activity 4

Step 1 - Take another piece of paper and fold it twice so that you obtain a right angled corner.

Step 2 - Now draw a figure on this paper so that it includes the right angled corner and such that it is limited to the region where the four portions of paper overlap.

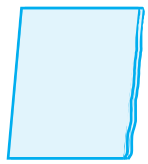
Cut the figure and unfold it. You will obtain a bilaterally symmetric figure with two axes of symmetry, where the axes of symmetry are the two lines along which you folded the paper.



Step 3 - Cut out other symmetric figures in this manner.

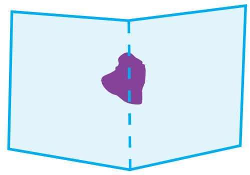
Activity 5

Step 1 - Get a paper and some paint.



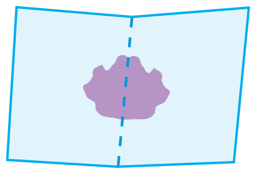
Step 2 - Fold the paper into two parts.

Step 3 - Now unfold the paper. On the side that is folded in, place a drop of paint so that it lies on the line of folding.



Step 4 - Now fold the paper back and press with your hand.

Step 5 - Unfold the paper.



At the end of this activity you will obtain a bilaterally symmetric figure as in the given diagram.

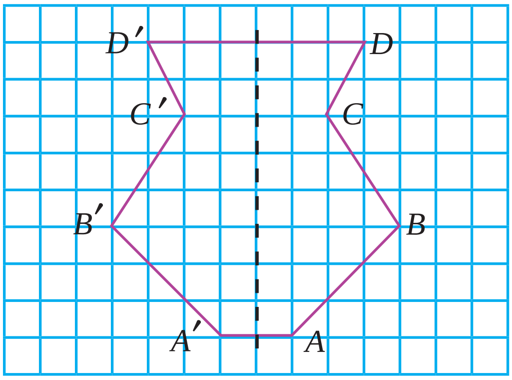
Step 6 - Following the above steps, obtain different bilaterally symmetric figures by changing the amount of paint used or by pressing down in different directions.

Assignment

- Create various bilaterally symmetric plane figures by cutting out folded paper as well as by placing drops of paint on folded paper as done in the previous activities.
- Prepare an attractive wall decoration using the symmetric figures that you created.

1.4 Drawing bilaterally symmetric plane figures

Let us consider the symmetric plane figure given below which has been drawn on a square ruled paper.



The axis of symmetry of this figure is the vertical line indicated by the dotted line. The points at which the straight line segments of a rectilinear plane figure meet are defined as the **vertices** of the plane figure. Usually the vertices are named using capital letters of the English alphabet.

The vertices A, B, C and D are on the right side of the axis of symmetry of the figure. Let us consider where the points A', B', C' and D' are located on the left side of the axis of symmetry.

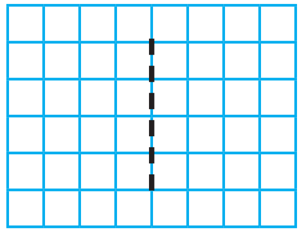
The point A' is located at a distance from the axis of symmetry which is equal to the distance from A to the axis of symmetry, on a horizontal line which passes through A. A' is defined as **the vertex corresponding to** A.

Similarly, B', C' and D' are defined as the vertices corresponding to B, C and D respectively.

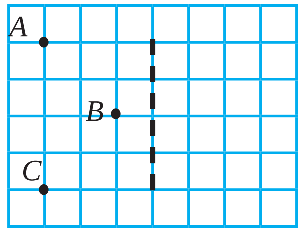
Let us consider how a bilaterally symmetric figure is drawn on a square ruled paper (or grid) by identifying corresponding vertices.

Activity 6

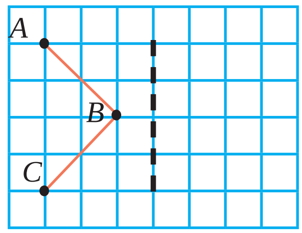
Step 1 - As indicated in the figure, select a vertical line on the grid and mark it with a dotted line.



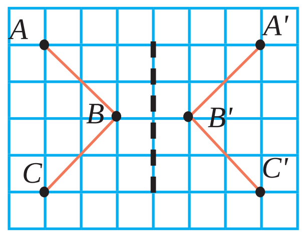
Step 2 - Select three points of intersection of vertical and horizontal lines on the grid which lie on the left side of the dotted line. Name these points as A, B and C respectively.



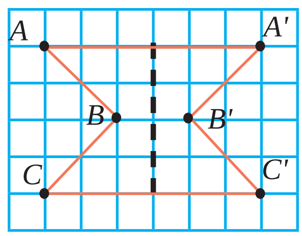
Step 3 - Join the points A and B, and the points B and C using straight line segments.



Step 4 - On the right side of the dotted line, mark on the grid, the points corresponding to the above points. Name them A', B' and C'. Join the points A' and B', and the points B' and C' using straight line segments.



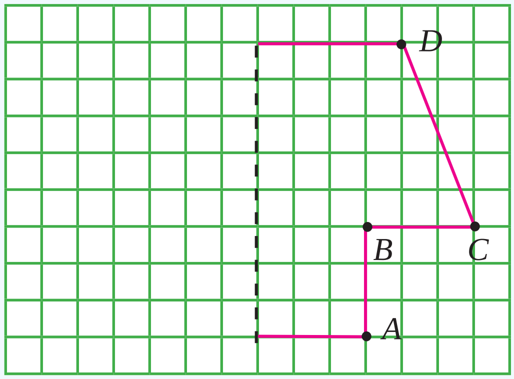
Step 5 - Join the points A and A', and the points C and C' using straight line segments.



Now you have obtained a bilaterally symmetric rectilinear plane figure with the dotted line as its axis of symmetry and the marked points as vertices.

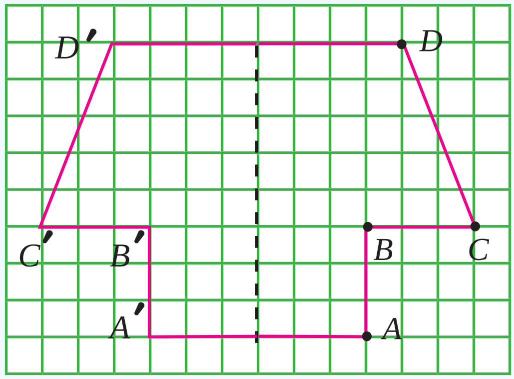
Let us consider how symmetric figures can be drawn by using the above properties.

Example 1



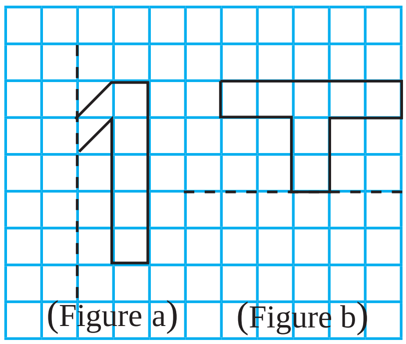
Complete the bilaterally symmetric figure such that the dotted line in the diagram is its axis of symmetry.

The distance from A and B to the axis of symmetry is equal to the length of three squares.



Therefore, let us mark the points A' and B' such that the distance from it to the axis of symmetry is also equal to the length of three squares, By similarly marking the points C' and D' such that the distance from C' to the axis of symmetry is equal to the length of 6 squares, and the distance from D' to the axis of symmetry is equal to the length of 4 squares as shown in the figure, and joining the points as indicated, we obtain a bilaterally symmetric figure.

Exercise 1.2



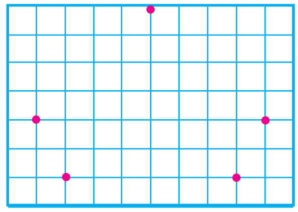
(1) (i) Copy figure *a* in your square ruled exercise book.

(ii) The dotted line indicates the axis of symmetry. Place a mirror on this line and observe the bilaterally symmetric figure.

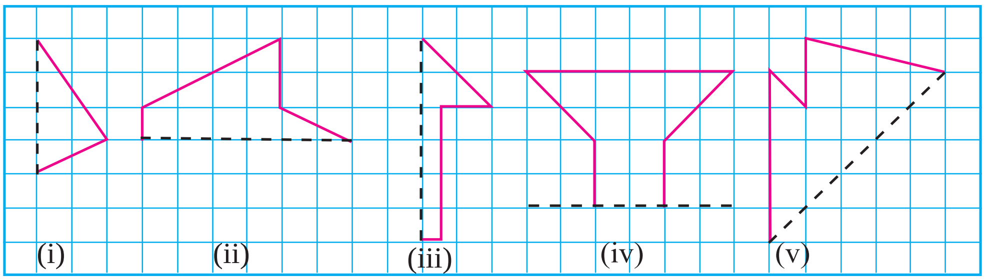
(iii) Draw and complete the bilaterally symmetric figure.

(iv) Repeat the above steps for figure *b* and complete the bilaterally symmetric figure.

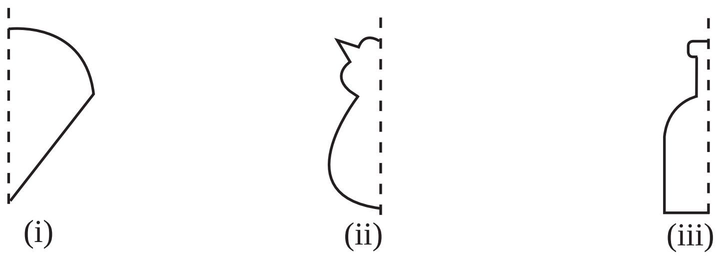
(2) Draw a bilaterally symmetric figure with the points marked on the grid as vertices and identify its axis of symmetry.



(3) Copy each of the figures given below in your exercise book. Complete the figures so that you obtain a bilaterally symmetric figure in each case.



(4) Trace each of the figures given below on a tissue paper and copy them in your exercise book.



Now turn the tissue paper on the dotted line. Draw the other half of each of the figures to obtain bilaterally symmetric figures.

(5) (i) Draw three bilaterally symmetric figures on a square ruled paper such that each figure has only one axis of symmetry.

(ii) Draw the axis of symmetry of each of the above figures.

(6) (i) Draw two bilaterally symmetric figures on a square ruled paper such that each figure has only 2 axes of symmetry.

(ii) Draw the axes of symmetry of each figure.

Summary

- If a plane figure is divided into two equal parts which coincide with each other when folded along a particular line, then that figure is defined as a bilaterally symmetric figure.
- The line of folding described above is an axis of symmetry of the figure.

2 Sets

By studying this lesson you will be able to

- identify sets,
- identify the elements of a set,
- write a set by listing the elements that belong to the set,
- write a set in terms of a common property of the elements of the set so that the elements can be clearly identified, and
- represent a set by a Venn diagram.

2.1 Introduction to Sets

The figure shows the types of vegetables that a certain vendor has for sale. The only types of vegetables that the vendor has are carrots, beans, pumpkins and ladies fingers. Accordingly, we can state with certainty whether the vendor has a certain type of vegetable for sale or not.

What has been given above is a collection of several items. Such a collection can be called a group. In our day to day life we have to make decisions on groups, that is, on such collections of items.

Let us consider the following groups.

- The districts that belong to the Southern Province of Sri Lanka
- The odd numbers between 1 and 10
- The vowels in the English alphabet
- The types of birds that are endemic to Sri Lanka, that have been identified by the year 2014
- The students who sat the Grade Five Scholarship Examination in 2014

The items that belong to these groups too can be clearly identified.

A group consisting of such items that can be clearly identified is called a **set**.

Various types of items can belong to a set. Numbers, physical objects, living beings and symbols too can belong to a set. A set can be expressed by writing down all the items in a certain group or by giving a common property or several common properties by which the items in the group can be clearly identified.

It can be stated with certainty whether a particular item belongs or does not belong to a set which has thus been specified.

The items that belong to a set are defined as its elements.

Accordingly, the district of Galle belongs to the set consisting of the districts of the Southern Province, while neither the district of Gampaha nor the district of Kalutara belongs to this set.

Three more examples of sets are given below.

- The set consisting of the even numbers between 1 and 10
- The set consisting of the symbols a, d, g, 5, 2
- The set consisting of the vehicles that were registered in Sri Lanka in 2014

The elements that belong to the above sets can be clearly identified.

Let us now consider the following.

- The tall students in a class
- Popular singers of Sri Lanka

The items that belong to such groups cannot be clearly identified since the common properties given above are subjective and debatable.

Therefore a set cannot be identified by considering such properties.

Exercise 2.1

(1) Place a ✓ next to each of the expressions which clearly define a set, and a × next to those which do not clearly define a set.

(i) Those who obtained more than 100 marks in the Grade 5 Scholarship examination held in 2013
(ii) Talented singers
(iii) Districts of Sri Lanka
(iv) Beautiful flowers
(v) Numbers between 0 and 50 which are multiples of 6
(vi) People who are fortunate

2.2 Writing a set

Let us now learn two methods of writing a set.

- **Writing a set by listing the elements of the set within curly brackets**

A set can be expressed by writing the elements of the set separated by commas, within curly brackets, when it is possible to list all the elements of the set.

The set consisting of the elements 9, 1, 3 is written as {9, 1, 3}.

➤ When writing a set in this form, the order in which the elements appear within the curly brackets is not important.

Thus, the above set can be written as {1, 3, 9} or {9, 3, 1} or {1, 9, 3} etc.

The set consisting of the elements a, b, d, 9, 1, 3 can be written as {1, 3, 9, a, b, d} or {1, a, 3, b, 9, d} or {a, 1, 3, b, 9, d} etc.

➤ Capital letters of the English alphabet are usually used to name sets.

Let A be the set of even numbers between 0 and 10. Then it can be written as follows. $A = \{2, 4, 6, 8\}$

Let B be the set of letters of the word "integers". Let us express B by writing its elements within curly brackets.

B = {i, n, t, e, g, r, s}.

Here the element “e” is written just once.

That is, even if an element appears several times within a group, it is written only once when it is written as an element of a set.

- **Writing a set by specifying common properties of its elements by which the elements of the set can be clearly identified**

A set can be expressed by writing a common property or common properties of the elements within curly brackets.

The set consisting of the even numbers between 1 and 10 can be written as {Even numbers between 1 and 10}.

The set consisting of the types of birds endemic to Sri Lanka that have been identified by the year 2014 can be written as {Types of birds endemic to Sri Lanka that have been identified by the year 2014}.

Since there are a large number of such types of birds, it is difficult to write this set by listing all the different types within curly brackets.

The set consisting of all odd numbers greater than 0, can be written as{Odd numbers greater than 0}.

Although this set cannot be expressed by writing down all its elements within curly brackets, it can be written as {1, 3, 5, 7, …}

If the elements of a set are in a certain order, when writing the set, the first few elements can be written, and to indicate the remaining elements an ellipsis (three periods) can be used within the curly brackets, after the first few elements.

Accordingly, the set of positive integers can be written as {1, 2, 3, 4, … }.

The set consisting of the types of birds endemic to Sri Lanka that have been identified by 2014 cannot be written in this manner.

Example 1

(i) Write the set A = {Prime numbers between 0 and 15} by writing all the elements that belong to A within curly brackets.
(ii) Are 1 and 17 elements of the set A?

(i) $A = \{2, 3, 5, 7, 11, 13\}$

(ii) Since 1 is not a prime number and 17 is a prime number which is greater than 15, they do not belong to A. Therefore they are not elements of A.

Example 2

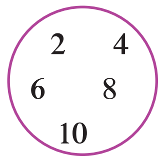
B = {The positive integers that are multiples of 3}. Write the elements of B within curly brackets.

$B = \{3, 6, 9, 12, 15, 18, ...\}$

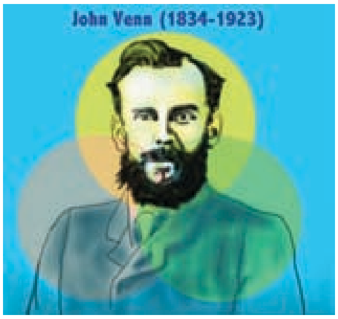
2.3 Representing a set by a Venn diagram

Let us write down the elements of the set A = {Even numbers from 1 to 10}.
$A = \{2, 4, 6, 8, 10\}$.

Let us represent this set by a closed figure as shown.

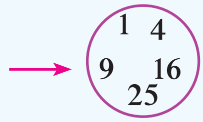
$A \longrightarrow$ 

When a set is represented in the above manner by a closed figure, such a figure is defined as a **Venn diagram.** The elements of the set are written inside the closed figure. Expressing a set in this manner as a closed figure is defined as, **representing a set by a Venn diagram.**



This method of representing a set by a figure was introduced by the English mathematician **John Venn**. Therefore such a closed figure is called a **Venn diagram**.

Example 1

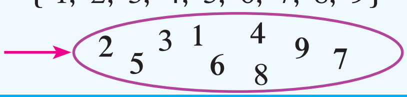
A set P has been represented here by a Venn diagram. P 

(i) Write down the set P by writing its elements within curly brackets.
(ii) Write P in terms of a common property by which the elements of P can be clearly identified.

(i) $P = \{1, 4, 9, 16, 25\}$
(ii) P = {Square numbers from 1 to 25}

Example 2

A is the set of positive whole numbers from 1 to 9.
(i) Write down the set A in terms of a common property of its elements.
(ii) Write down the set A by listing its elements.
(iii) Represent the set A by a Venn diagram.

(i) A = { Positive whole numbers from 1 to 9}
(ii) $A = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
(iii) A 

Exercise 2.2

(1) (a) Express each of the following sets by writing all the elements of each set within curly brackets.
(i) A = {Days of the week}
(ii) B = {Prime numbers between 0 and 10}
(iii) C = {Multiples of 4 between 0 and 25}
(iv) D = {Letters of the word "diagram"}
(v) E = {Districts of the western province}
(vi) F = {Digits of the number 21 412 }
(vii) G = {Multiples of 6 from 1 to 10}

(b) For the sets defined above, state whether the following statements are true or false.
(i) "Saturday" is an element of A. (ii) "p" is an element of D.
(iii) All the elements of C are even numbers.
(iv) Any multiple of 3 from 1 to 10 is an element of G.

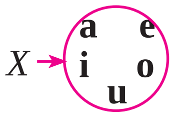
(2) Express each of the following sets in a different form by writing all the elements of each set within curly brackets.
Represent each of these sets by a Venn diagram too.

(i) P = {Prime numbers less than 10}
(ii) Q = {Colours of a rainbow}
(iii) R = {Letters of the word "number"}
(iv) S = {Whole numbers between 0 and 7}
(v) T = {Districts of the Southern Province}

(3) $K = \{4, 8, 12, 16, 20\}$

(i) Represent the set K by a Venn diagram.
(ii) Write down the set K in terms of a common property of its elements by which the elements can be clearly identified.

(4) The set X has been represented by a Venn diagram here.



(i) Express the set X in a different form by writing the elements of X within curly brackets.
(ii) Write down the set X in terms of a common property of its elements by which the elements can be clearly identified.

(5) Represent the set of multiples of 5 between 6 and 25.

(i) by writing down a common property by which the elements of the set can be clearly identified,
(ii) by writing all the elements of the set within curly brackets,
(iii) by a Venn diagram.

Summary

- A group of items that can be clearly identified is defined as a set.
- The items in a set are called its elements.
- A set can be expressed by writing the elements of the set separated by commas within curly brackets.
- An element of a set is written just once when the set is expressed in terms of its elements.
- A set can be expressed by writing a common property or common properties of the elements by which the elements can be clearly identified, within curly brackets.
- A set can be represented by a Venn diagram.

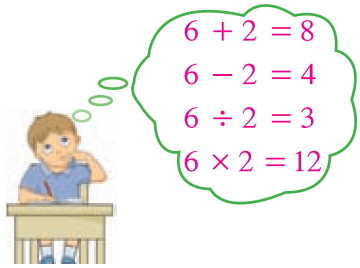
3

Operations on whole numbers

By studying this lesson you will be able to
- identify the convention used in simplifying numerical expressions, and
- simplify numerical expressions consisting of whole numbers.

3.1 Mathematical operations on two whole numbers

Addition, multiplication, subtraction and division are symbolized by +, ×, – and ÷ respectively.



You have already learnt how to add and multiply two whole numbers.

Further, you know how to subtract one whole number from another, and how to divide one whole number by another.

Here each mathematical operation was performed only once.

3.2 The order in which mathematical operations in a numerical expression are performed

Consider the expression $3 + 7 \times 5$.

This is a numerical expression with three whole numbers and two operations.

Here + and × are defined as the operations of this expression.
The order in which the operations appear is + first and then ×.

If we consider the expression $15 \div 3 - 2$, the order in which the operations appear is ÷ first and then –.

Example 1

Write down the operations of the expression $12 \times 2 - 5 \times 3$ in the order in which they appear.

The order in which the operations appear is $\times$, $-$ and $\times$.

Exercise 3.1

(1) For each of the following numerical expressions, write down the mathematical operations in the order in which they appear.

(i) $5 + 3 + 2$ (ii) $6 \times 3 - 6$ (iii) $10 - 8 \div 2 \times 3$

(iv) $11 \times 2 + 5 - 2$ (v) $24 \div 6 + 6 \div 3$

3.3 Simplifying numerical expressions

• Simplifying expressions involving only addition

Let us simplify the expression $8 + 7 + 2$ in two different ways.

Let us add 8 and 7 first, and then add 2 to the result. This yields the answer 17.

$8 + 7 + 2 = 15 + 2 = 17$

Adding 7 and 2 first, followed by adding 8 to the result also yields the answer 17.

$8 + 7 + 2 = 8 + 9 = 17$

• Simplifying expressions involving only multiplication

Let us simplify the expression $5 \times 2 \times 3$ in two different ways.

Multiplying 5 by 2 first, and then multiplying the result by 3 yields the answer 30. $5 \times 2 \times 3 = 10 \times 3 = 30$

Multiplying 2 by 3 first, and then multiplying the result by 5 also yields the answer 30.

$5 \times 2 \times 3 = 5 \times 6 = 30$

Thus, if either addition or multiplication is the only operation in a numerical expression, then irrespective of the order in which the operations are performed, the result obtained is the same.

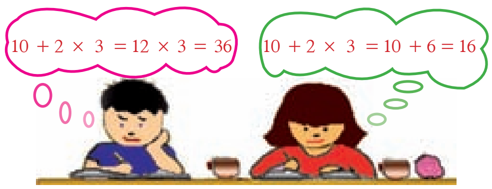
Exercise 3.2

(1) Simplify each of the following expressions.

(i) $12 + 5 + 8$ (ii) $5 \times 8 \times 3$ (iii) $7 + 3 + 2 + 6$ (iv) $2 \times 5 \times 4 \times 3$

3.4 Further simplification of numerical expressions

Let us simplify the expression $10 + 2 \times 3$. Let us compare the answers we obtain when we simplify $10 + 2 \times 3$ by performing the operations in two different orders.



Let us first add 10 to 2 and then multiply the answer by 3.
$10 + 2 \times 3 = 12 \times 3 = 36.$

Now let us multiply 2 by 3, and then add 10 to it.
$10 + 2 \times 3 = 10 + 6 = 16$

Therefore, it is clear that when numbers simplify such numerical expressions which involve more than two terms and several operations, we may end up with different answers, depending on the order in which we perform the operations.

This emphasises the need for a convention when simplifying expressions involving two or more operations.

Let us consider below the convention used when simplifying such expressions.

- First perform all divisions (÷) and multiplications (×), working from left to right.
- Then perform all additions (+) and subtractions (–), working from left to right.

Only the operations of addition and multiplication appear in the expression $10+2\times3$. According to the above convention, multiplication should be performed first.

$10 + 2 \times 3 = 10 + 6 = 16$

Also, if only subtraction (–) and addition (+), or only division (÷) and multiplication (×) appear in a numerical expression, simplification is done from left to right in the order that the operations appear.

➤ Simplifying expressions involving only addition and subtraction

Let us simplify the expression $10 - 7 + 2$.

Here the order in which the operations appear from left to right is – first and then +.

When simplifying 10 – 7 + 2, first 7 is subtracted from 10 and then 2 is added to the result.

$\therefore\ 10 - 7 + 2 = 3 + 2 = 5$

Another example is: $6 + 7 - 2 = 13 - 2 = 11$

➤ Simplifying expressions involving only multiplication and division

Let us deal with the expression 36 ÷ 6 × 3 in a similar way.
Here the order in which the operations appear from left to right is ÷ first and then ×.

Let us first divide 36 by 6, and then multiply the answer by 3.
We then obtain $36 \div 6 \times 3 = 6 \times 3 = 18$

Another example is: $36 \times 6 \div 3 = 216 \div 3 = 72$

➤ Simplifying expressions in which only subtraction (–) or division (÷) appears several times.

When simplifying expressions involving only subtraction (–) or division (÷), the order in which the operations are performed is from left to right.

Consider the expression 10 – 3 – 2, where the operation of subtraction is applied twice. When the expression 36 ÷ 6 ÷ 3 is considered, division is applied twice.

Let us simplify these expressions.

Now let us subtract 3 from 10, and then subtract 2 from the result.
10 – 3 – 2 = 7 – 2 = 5.
Let us divide 36 by 6, and then divide the answer by 3.We then obtain,
36 ÷ 6 ÷ 3 = 6 ÷ 3 = 2.

Example 1

Simplify $7 - 4 + 5$.

$7 - 4 + 5 = 3 + 5$
$= 8$

Example 2

Simplify $80 \div 10 \times 5$.

$80 \div 10 \times 5 = 8 \times 5$
$= 40$

Example 3

Simplify $4 \times 6 \div 3$.

$4 \times 6 \div 3 = 24 \div 3$
$= 8$

Example 4

Simplify $25 + 10 - 7$.

$25 + 10 - 7 = 35 - 7$
$= 28$

Example 5

Simplify $28 \div 2 - 3$.

$28 \div 2 - 3 = 14 - 3$
$= 11$

Example 6

Simplify $50 - 10 \times 3$.

$50 - 10 \times 3 = 50 - 30$
$= 20$

Example 7

Simplify $18 \times 5 - 62$.

$18 \times 5 - 62 = 90 - 62$
$= 28$

Example 8

Simplify $50 - 10 \div 2$.

$50 - 10 \div 2 = 50 - 5$
$= 45$

Example 9

Simplify $5 + 6 \div 3 + 2$.

$5 + 6 \div 3 + 2 = 5 + 2 + 2$
$= 9$

Example 10

Simplify $2 \times 12 \div 3 \times 5$.

$2 \times 12 \div 3 \times 5 = 24 \div 3 \times 5$
$= 8 \times 5 = 40$

Exercise 3.3

(1) Place a ✓ next to the correct statements and a × next to the incorrect statements.

(i) $8 - 5 + 2 = 1$ (ii) $12 \times 3 - 11 = 25$

(iii) $7 + 18 \div 6 = 10$ (iv) $5 \times 6 \div 3 + 7 = 3$

(2) Simplify the following expressions.

(i) $10 \times 4 + 17$ (ii) $8 \times 3 + 5$ (iii) $14 \div 7 \times 5$

(iv) $448 + 12 \div 3$ (v) $7 \times 200 + 108$ (vi) $8 \times 9 - 61$

(vii) $100 - 7 \times 8$ (viii) $195 - 12 \times 10 \div 5$ (ix) $7 + 5 \times 37 + 278$

• Simplifying expressions with brackets

If we want to subtract 2 from 3 first, and then subtract the result from 10, we write it as $10 - (3 - 2)$, with $3 - 2$ within brackets. This emphasises that the operation within brackets has to be done first.

That is, $10 - (3 - 2) = 10 - 1 = 9$.

Consider the following example.

A practical examination in music is held over six days. Each day, twelve candidates participate in the morning session, while 8 participate in the afternoon session. Let us find the total number of candidates.

Number of candidates in each morning session $= 12$
Number of candidates in each afternoon session $= 8$
Total number of candidates during the six days $= (12 + 8) \times 6$
$= 20 \times 6 = 120$

Observe that the usage of brackets has been necessary in deriving the correct answer.

The convention followed when simplifying expressions involving whole numbers and the operations $+, -, \times, \div$, and brackets is as follows.

- First perform any calculations inside brackets.
- Then perform all multiplications and divisions, working from left to right.
- Finally perform all additions and subtractions, working from left to right.

Example 1

Simplify $20 \div (12 - 7)$.

$20 \div (12 - 7) = 20 \div 5 = 4$

Example 2

Simplify $5 \times (10 + 12) \div 11$.

$$5 \times (10 + 12) \div 11 = 5 \times 22 \div 11$$
$$= 110 \div 11 = 10$$

Example 3

Simplify $8 + 5 \times (10 + 2) \div 3 - 4$

$$\begin{aligned} 8 + 5 \times (10 + 2) \div 3 - 4 &= 8 + 5 \times 12 \div 3 - 4 \\ &= 8 + 60 \div 3 - 4 \\ &= 8 + 20 - 4 \\ &= 28 - 4 \\ &= 24 \end{aligned}$$

Example 4

The pencils in five boxes, each of which contains 12 pencils, are divided equally among 4 students. Write down an expression for the number of pencils a single student receives, and simplify it.

$(12 \times 5) \div 4 = 60 \div 4 = 15$

The number of pencils each child receives is 15.

Example 5

Nimal plucked 47 mangoes from a tree in his garden. He kept 18 in his possession, and sold the rest to his neighbour at Rs. 9 per fruit. Write down an expression for the total amount of money Nimal earned in rupees by selling the mangoes, and simplify it.

$(47 - 18) \times 9 = 29 \times 9 = 261$

This can also be written as $9 \times (47 - 18)$ or as $9\ (47 - 18)$, omitting the multiplication symbol.

The total amount earned by selling the mangoes is 261 rupees.

Example 6

The taxi fare for the first kilometre is Rs. 50. It is Rs. 42 for each kilometre above the first. Write down an expression for the amount paid by a passenger who enjoyed a ride of 12 kilometres. Simplify your expression.

$50 + 42\ (12 - 1) = 50 + 42 \times 11 = 50 + 462 = 512$

The total amount paid is 512 rupees.

Exercise 3.4

(1) Simplify the following expressions.

(i) $(12 + 8) - 15$ (ii) $35 - (14 + 9)$ (iii) $7\ (12 - 7)$

(iv) $108 + 3\ (27 - 13)$ (v) $24 \div (17 - 5)$ (vi) $3\ (5 + 2) \times 8$

(vii) $31 + (16 \div 4)$ (viii) $73 - (8 \times 9)$ (ix) $(19 \times 10) + 38$

(x) $475 - (30 \div 6)$

(2) An international call to a certain country costs Rs. 7 for the first minute, and Rs. 4 per minute thereafter. Write down an expression in rupees for the cost of a 10 minutes long international call. Simplify your expression.

(3) Write down an expression for the number of two-litre bottles that can be filled with a fruit drink made from 8 litres of water and 4 litres of fruit juice.

(4) Simplify the following expressions.

(i) $30 \div 10 \times 5$ (ii) $40 \times 10 \div 5$ (iii) $400 - 20 \times 10$

(iv) $30 \div (10 \times 3)$ (v) $(40 \div 10) \times 8$ (vi) $3 + 7 \times 5$

(vii) $6 \div 2 + 7$ (viii) $(24 \times 3) \div 8$ (ix) $24 \div (3 \times 4)$

(x) $3 + 6 \times (5 + 4) \div 3 - 7$ (xi) $10 + 8\ (11 - 3) \times 4 - 4$

Summary

- The convention followed when simplifying expressions involving whole numbers and the operations $+$, $-$, $\times$, $\div$ and brackets is as follows.
  - First perform any calculations inside brackets.
  - Then perform all multiplications and divisions, working from left to right.
  - Finally perform all additions and subtractions, working from left to right.

4 Factors and Multiples
(Part I)

By studying this lesson, you will be able to
- examine whether a whole number is divisible by 3, 4, 6 or 9.

4.1 Examining whether a number is divisible by 3, 4, 6 or 9

It is important to know the divisibility rules when solving problems related to factors and multiples.

If a certain whole number can be divided by another whole number without remainder, then the first number is said to be divisible by the second number. We then identify the second number as a factor of the first number.

$6 \div 2 = 3$ with remainder 0. Therefore, 2 is a factor of 6.

$6 \div 4 = 1$ with remainder 2. Therefore, 4 is not a factor of 6.

One way to find factors of numbers quickly is to use tests of divisibility.

The divisibility rules you learnt in grade 6 are as follows.

- If the ones place digit of a number is divisible by 2, then that number is divisible by 2.
- If the ones place digit of a number is either 0 or 5, then that number is divisible by 5.
- If the ones place digit of a number is 0, then that number is divisible by 10.

• The digital root

Now let us learn about the digital root of a number.

The digital root of a number is calculated by adding up all the digits of that number (and adding the digits of the sums if necessary), until a single digit from 1 to 9 is left. That single digit is defined as the **digital root** of the relevant number.

Let us see how the digital root of a number is found by considering the following example.

Let us find the digital root of 213. Let us add the digits of 213.

$2 + 1 + 3 = 6$

Then the digital root of 213 is 6.

The digital root of 242 = $2 + 4 + 2 = 8$

Let us find the digital root of 68.

$6 + 8 = 14$. Let us add the digits of 14. $1 + 4 = 5$.

$\therefore$ The digital root of 68 is 5.

It is possible to identify certain properties of a number by considering its digital root.

- **Examining whether a number is divisible by 9**

Let us do the following activity. Our goal is to identify a rule to examine whether a number is divisible by 9 or not.

Activity 1

Complete the following table and answer the given questions.

Number	Digital root	Remainder when divided by 9	Is the number divisible by 9?	Is 9 a factor?
45				
52				
134				
549				
1323				
1254				
5307				

(i) What are the digital roots of the numbers which are divisible by 9, in other words, the numbers of which 9 is a factor.

(ii) Using your answer to the previous part, suggest a method (other than division) to test whether a number is divisible by 9.

- If the digital root of a whole number is divisible by 9, then that number is divisible by 9. That is 9 is a factor of that number.

• Examining whether a number is divisible by 3

Let us do the following activity. Our goal is to identify a rule to examine whether a number is divisible by 3 or not.

Activity 2

Complete the following table and answer the given questions.

Number	Digital root of the number	Is the digital root of the number divisible by 3?	Is the number divisible by 3?	Is 3 a factor?
15				
16				
24				
28				
210				
241				
372				
1269				

(i) What values do you get as the digital roots of the numbers which are divisible by 3, in other words, the numbers of which 3 is a factor?

(ii) Does 3 always divide the digital root of the numbers which are divisible by 3?

(iii) Is every number of which the digital root is indivisible by 3, indivisible by 3?

If the digital root of a whole number is divisible by 3, then that number is divisible by 3. That is 3 is a factor of that number.

Exercise 4.1

(1) Without dividing, select the numbers which are divisible by 9.

504, 652, 567, 856, 1143, 1351, 2719, 4536

(2) Without dividing, select and write down the numbers which are divisible by 3.

81, 102, 164, 189, 352, 372, 466, 756, 951, 1029

(3) 3 divides the number 65☐. Suggest two digits suitable for the empty space.

(4) Pencils were brought to be distributed among Nimal's friends on his birthday Party. The number of pencils was less than 150, but close to it. Nimal observed that each friend could be given 9 pencils. What is the maximum number of pencils that may have been brought?

(5) The following quantities of items were brought to make gift packs to be given to the winners of a competition.

131 exercise books	130 pencils
128 platignum pens	131 ballpoint pens

If each gift pack should contain 3 units of each item, write down the minimum extra amounts needed from each item.

• Examining whether a number is divisible by 6

You have learnt previously that, if the ones place digit of a number is zero or an even number, then that number is divisible by 2. You have also learnt how to determine whether a number is divisible by 3. Do the following activity to examine whether a number is divisible by 6.

Activity 3

Complete the following table and answer the given questions.

Number	Is the number divisible by 2 ?	Is the number divisible by 3 ?	Is the number divisible by 6 ?	Is 6 a factor?
95				
252				
506				
432				
552				
1236				

(i) Are all numbers which are divisible by 6, divisible by 2 also?
(ii) Are all numbers which are divisible by 6, divisible by 3 also?
(iii) Are all numbers which are divisible by 6, divisible by both 2 and 3?

(iv) Suggest a suitable method to identify the numbers which are divisible by 6, in other words, the numbers of which 6 is a factor.

If a number is divisible by both 2 and 3, then it is divisible by 6.

That is 6 is a factor of that number.

• Examining whether a number is divisible by 4

In order to identify a rule to determine whether a number is divisible by 4, do the following activity.

Activity 4

Complete the following table and answer the given questions.

Number	Is the ones place digit divisible by 4?	Is the number formed by the last two digits divisible by 4?	Is the number divisible by 4?	Is 4 a factor?
36				
259				
244				
600				
658				
1272				
4828				

(i) Is the ones place digit of every number which is divisible by 4, divisible by 4?

(ii) Is the digital root of every number which is divisible by 4, divisible by 4?

(iii) Which of the above properties should be used to determine whether a number is divisible by 4?

If the last two digits of a whole number consisting of two or more digits is divisible by 4, then that number is divisible by 4. That is 4 is a factor of that number.

Exercise 4.2

(1) From the following, select and write down the numbers

(i) Which are divisible by 6.
(ii) Which are divisible by 4.

162, 187, 912, 966, 2118, 2123, 2472, 2541, 3024, 3308, 3332, 4800

(2) Write the following numbers in the appropriate column of the table given below. (A number may be written in both column (i) and column (iii).)

348, 496, 288, 414, 1024, 1272, 306, 258, 1008, 6700

(i)	(ii)	(iii)	(iv)
Numbers of which 4 is a factor	Reason for your selection	Numbers of which 6 is a factor	Reason for your selection

(3) The number 62□6 is divisible by both 4 and 6. Find the suitable digit for the empty space.

(4) A drill team arranges themselves in the following manner. On one occasion they form lines consisting of 3 members each and on another occasion lines consisting of 4 members each. They also make circles of 9 members each. If the drill team must have more than 250 members, use the divisibility rules to find the minimum number of members that could be in the team.

(5) Determine whether 126 is divisible by 2, 3, 4, 5, 6, 9 and 10.

Summary

Number	Divisibility Rule
2	If 2 divides the ones place digit of a whole number, then that number is divisible by 2.
3	If the digital root of a whole number is divisible by 3, then that number is divisible by 3.
4	If the last two digits of a whole number consisting of two or more digits is divisible by 4, then that whole number is divisible by 4.
5	If the ones place digit of a whole number is either 0 or 5, then that number is divisible by 5.
6	If a whole number is divisible by both 2 and 3, then it is divisible by 6.
9	If the digital root of a whole number is 9, then that number is divisible by 9.
10	If the ones place digit of a whole number is 0, then that number is divisible by 10.

4 Factors and Multiples
(Part II)

By studying this lesson, you will be able to
- find the factors of a whole number,
- write the multiples of a whole number,
- write the prime factors of a whole number,
- find the highest common factor (HCF) of a whole number and
- find the least common multiple (LCM) of a whole number.

4.2 Factors and multiples of a whole number

You learnt in grade six, how to find the factors and multiples of a whole number. Let us recall what you learnt.

Let us find the factors of 36.

Let us factorize 36 by expressing it as a product of two whole numbers.

$36 = 1 \times 36$
$36 = 2 \times 18$
$36 = 3 \times 12$
$36 = 4 \times 9$
$36 = 6 \times 6$

When a whole number is written as a product of two whole numbers, those two numbers are known as **factors** of the original number.

Therefore, the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Let us factorize 126, using the method of division.

$$\begin{array}{r|l} 2 & 126 \\ \hline & \;\;63 \end{array}$$

Since the number 126, can be divided by 2 without remainder, 2 is a factor of 126.
Since, $2 \times 63 = 126$, we obtain that 63 is also a factor of 126.

$$\begin{array}{r|l} 3 & 126 \\ \hline & \;\;42 \end{array} \quad \begin{array}{r|l} 6 & 126 \\ \hline & \;\;21 \end{array} \quad \begin{array}{r|l} 7 & 126 \\ \hline & \;\;18 \end{array} \quad \begin{array}{r|l} 9 & 126 \\ \hline & \;\;14 \end{array} \quad \begin{array}{r|l} 14 & 126 \\ \hline & \;\;\;\;9 \end{array}$$

$2 \times 63 = 126$ $\quad$ $3 \times 42 = 126$ $\quad$ $6 \times 21 = 126$
$7 \times 18 = 126$ $\quad$ $9 \times 14 = 126$ $\quad$ $14 \times 9 = 126$

Therefore, the factors of 126 are 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63 and 126.

Note:
The divisibility rules can be used to determine whether a given number is divisible by another number or not.

Now let us consider how multiples of a whole number are found.

Let us compute the multiples of 13.
This can be done by multiplying 13 by whole numbers.

$13 \times 1 = 13$ $13 \times 2 = 26$ $13 \times 3 = 39$ $13 \times 4 = 52$

13, 26, 39 and 52 are a few examples of multiples of 13. Note that 13 is a factor of each of them. Therefore, any number of which 13 is a factor, is a multiple of 13.

Exercise 4.3

(1) Factorize.
(i) 150 (ii) 204 (iii) 165 (iv) 284

(2) Write down the ten factors of 770 below 100.

(3) (i) Write five multiples of 36.
(ii) Write five multiples of 112.
(iii) Write five multiples of 53 below 500.

(4) 180 chairs in an examination hall have to be arranged such that each row has an equal number of chairs. If the minimum number of chairs that should be in a row is 10 and the maximum that could be in a row is 15, find how many possible ways there are to arrange the chairs.

4.3 Prime factors of a whole number

You have already learnt that whole numbers greater than one with exactly two distinct factors are called prime numbers.

Let us recall the prime numbers below 20.

They are 2, 3, 5, 7, 11, 13, 17 and 19.

Let us identify the prime factors of 36. We learnt above that the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

There are only two prime numbers among them, namely, 2 and 3. These are the prime factors of 36.

Let us find the prime factors of 60.
The factors of 60 are 1, 2, 3, 4, 5, 6, 12, 15, 20, 30 and 60.
The prime factors among them are 2, 3 and 5.

The prime numbers among the factors of a number are its prime factors.

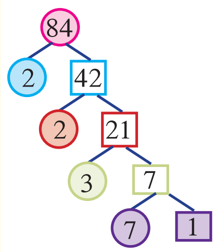
Any whole number which is not a prime number can be expressed as a product of its prime factors.

A method of finding the prime factors of a whole number using the method of division and writing the number as a product of its prime factors is described below.

Let us find the prime factors of 84 and write it as a product of its prime factors.

- Here 84 has been divided by 2, the smallest prime number.
- Division by 2 is continued, until a number which is not divisible by 2 is obtained.
- When this result is divided by the next smallest prime, which is 3, the result 7 is obtained. When this is divided by the prime number 7, the answer obtained is 1.
- In this manner, we continue dividing by prime numbers until the answer 1 is obtained.

$$\begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$



Accordingly, the prime factors of 84 are 2, 3 and 7, which are the numbers by which 84 was divided.

Now, to write 84 as a product of its prime factors, write it as a product of the prime numbers by which it was divided.

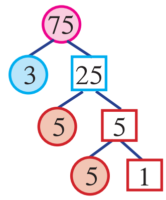
$84 = 2 \times 2 \times 3 \times 7$

Let us write 75 as a product of its prime factors. Let us divide 75 by prime numbers.

- Since 75 is not divisible by 2, we divide it by 3, the next smallest prime number.
- The result 25 is not divisible by 3.
- When 25 is divided twice by 5 which is the next smallest prime number, the result is 1.

Accordingly, when 75 is written as a product of its prime factors we obtain $75 = 3 \times 5 \times 5$.

$$\begin{array}{r|l} 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$



- When finding the prime factors of a whole number, the number is divided by the prime numbers which divide it without remainder, starting from the smallest such prime number, until the answer 1 is obtained.
- The prime numbers which divide a number without remainder are its prime factors.
- To write a number as a product of its prime factors, write it as a product of the prime numbers by which it was divided.

Example 1

Write 63 as a product of its prime factors.

$$\begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

Since 63 is not divisible by 2, division is started with 3. The result 21 is again divided by 3. Then we obtain 7 which is not divisible by 3. We divide this by 7 to obtain 1.

Therefore, 63 written as a product of its prime factors is

$63 = 3 \times 3 \times 7.$

Exercise 4.4

(1) Find the prime factors of each of the following numbers.

(i) 81 (ii) 84 (iii) 96

(2) Express each of the following numbers as a product of its prime factors.

(i) 12 (ii) 15 (iii) 16 (iv) 18 (v) 20
(vi) 28 (vii) 59 (viii) 65 (ix) 77 (x) 91

4.4 Finding the factors of a number by considering its prime factors

Suppose we need to find the factors of 72.
Let us start by writing 72 as a product of its prime factors.

$$
\begin{array}{r|l}
2 & 72 \\ \hline
2 & 36 \\ \hline
2 & 18 \\ \hline
3 & 9 \\ \hline
3 & 3 \\ \hline
 & 1
\end{array}
$$

$$
\begin{aligned}
72 &= 2 \times 2 \times 2 \times 3 \times 3 &&= 1 \times 72 \\
72 &= 2 \times (2 \times 2 \times 3 \times 3) &&= 2 \times 36 \\
72 &= (2 \times 2) \times (2 \times 3 \times 3) &&= 4 \times 18 \\
72 &= (2 \times 2 \times 2) \times (3 \times 3) &&= 8 \times 9 \\
72 &= (2 \times 2 \times 2 \times 3) \times 3 &&= 24 \times 3
\end{aligned}
$$

The factors of a whole number (which are not its prime factors or 1) can be obtained by taking products of 2 or more of its prime factors.

2, 36, 4, 18, 8, 9, 24 and 3 are eight factors of 72. 1 and 72 are also factors of 72.

1, 2, 3, 4, 8, 9, 18, 24, 36 and 72 are ten factors of 72.

Exercise 4.5

(1) Find six factors of each of the following numbers by considering their prime factors.

(i) 20 (ii) 42 (iii) 70 (iv) 84 (v) 66 (vi) 99

4.5 Highest Common Factor (HCF) (Greatest Common Divisor (GCD))

Let us now consider what the highest common factor (HCF) of several numbers is and how it is found.

Let us find the highest common factor of the numbers 6, 12 and 18.

- Write down the factors of these numbers as follows.

  Factors of 6: 1, 2, 3 and 6
  Factors of 12: 1, 2, 3, 4, 6 and 12
  Factors of 18: 1, 2, 3, 6, 9 and 18

- Circle and write the factors common to all three numbers.

  The factors which are common to 6, 12 and 18 are, 1, 2, 3 and 6.
- The largest number among the common factors is the **highest common factor** of these numbers.

We observe that the largest or the greatest of these common factors is 6. Therefore, 6 is the **highest common factor** of 6, 12 and 18.

Thus, the highest common factor of 6, 12 and 18 is 6, which is the largest number by which these three numbers are divisible.

- Given two or more numbers, the largest number which is a factor of all of them is known as their highest common factor (HCF).
- Accordingly, the highest common factor is the largest number by which all the given numbers are divisible.
- If 1 is the only common divisor of several numbers, then the highest common factor of these numbers is 1.

• Finding the highest common factor by writing each number as a product of its prime factors

Let us find the highest common factor of 6, 12 and 18.

- Let us write each number as a product of its prime factors.

$$6 = 2 \times 3$$
$$12 = 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$

$$\begin{array}{r|l} 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$6 = 2 \times 3$ $\quad 12 = 2 \times 2 \times 3$ $\quad 18 = 2 \times 3 \times 3$

- The highest common factor is obtained by taking the product of the prime factors which are common to all three numbers.

2 and 3 are the common prime factors of 6, 12 and 18.
Thus, the HCF of 6, 12 and 18 is 2 × 3 = 6.

• Finding the highest common factor by the method of division

Let us find the highest common factor of 6, 12 and 18.

$$\begin{array}{r|lll} 2 & 6, & 12, & 18 \\ \hline 3 & 3, & 6, & 9 \\ \hline & 1, & 2, & 3 \end{array}$$

- Write the numbers as shown.
- Since all these numbers are divisible by 2, divide each of them by 2 individually.
- The result is 3, 6 and 9. Since 3, 6 and 9 are divisible by 3, the next smallest prime number, divide them by 3 and write the result below the respective numbers.
- The result is 1, 2 and 3. Since there isn't a prime number which divides all of 1, 2 and 3 without remainder, the division is stopped here.
- The HCF is obtained by multiplying the numbers by which division was done.

Thus the HCF of 6, 12 and 18 is 2 × 3 = 6.

When using the method of division to find the HCF,
- keep dividing all the numbers by the prime numbers which divide all the numbers without remainder.
- then multiply all the divisors and obtain the HCF of the given numbers.

The HCF of any set of prime numbers is 1.

Example 1

Find the highest common factor of 72 and 108.

Method I

The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36 and 72.
The factors of 108 are 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54 and 108.

When the factors common to both these numbers are selected we obtain 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Since the greatest of these common factors is 36, the highest common factor of 72 and 108 is 36.

Method II

Let us write 72 and 108 as products of their prime factors.

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$72 \ = 2 \times 2 \times 2 \times 3 \times 3$
$108 = 2 \times 2 \times 3 \times 3 \times 3$

The prime factors which are common to both the numbers 72 and 108 are 2, 2, 3 and 3.

Accordingly, the HCF of 72 and 108 $\Big\} = 2 \times 2 \times 3 \times 3 = 36$

Method III

Find the HCF of 72 and 108.

$$\begin{array}{r|l} 2 & 72,\ 108 \\ \hline 2 & 36,\ \ 54 \\ \hline 3 & 18,\ \ 27 \\ \hline 3 & 6,\ \ \ 9 \\ \hline & 2,\ \ \ 3 \end{array}$$

Since there isn't another prime number which divides both 2 and 3 without remainder, stop the division here.

The HCF of 72 and 108 $\Big\} = 2 \times 2 \times 3 \times 3$
$= 36$

The HCF 36 of 72 and 108 can also be described as the largest number that divides them both without remainder.

Example 2

(1) Three types of items were brought in the following quantities to be offered at an almsgiving.

30 cakes of soap

24 tubes of toothpaste

18 bottles of ointment

Parcels were made such that each contained all three items. Every parcel had the same number of items of a particular type. What is the maximum number of parcels that can be made accordingly? Write down the quantity of each item in a single parcel.

Every parcel should contain the same number of items of a particular type. To find the maximum number of parcels that can be made, we need to find the largest number which divides 30, 24 and 18 without remainder.

Therefore, let us find the HCF of 30, 24 and 18.

$$30 = 2 \times 3 \times 5$$
$$24 = 2 \times 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$

HCF of 30, 24 and 18 $= 2 \times 3 = 6$.

The maximum number of parcels that can be made = 6

The number of cakes of soap in a parcel $= 30 \div 6 = 5$

The number of tubes of toothpaste in a parcel $= 24 \div 6 = 4$

The number of bottles of ointment in a parcel $= 18 \div 6 = 3$

Exercise 4.6

(1) Fill in the blanks to obtain the HCF by writing down all factors of the given numbers.

(i) Factors of 8 are ...., ..., ... and .....

Factors of 12 are ...., ..., ..., ..., ..., and .....

Factors common to 8 and 12 are ...., ..., and .....

$\therefore$ The HCF of 8 and 12 is .... .

(ii) 54 written as a product of its prime factors = 2 × ..... × 3 × .....
90 written as a product of its prime factors = ... × 3 × ..... × 5.
72 written as a product of its prime factors = 2 × 2 × ..... × ..... × .....
∴ The HCF of 54, 90 and 72 = ... × ..... × .....
= .....

(2) Find the HCF of each pair of numbers by writing down all their factors.

(i) 12, 15 (ii) 24, 30 (iii) 60, 72
(iv) 4, 5 (v) 72, 96 (vi) 54, 35

(3) Find the HCF of each pair of numbers by writing each number as a product of its prime factors.

(i) 24, 36 (ii) 45, 54 (iii) 32,48 (iv) 48, 72 (v) 18, 36

(4) Find the HCF by any method you like.

(i) 18, 12, 15 (ii) 12, 18, 24 (iii) 24, 32, 48 (iv) 18, 27, 36

(5) A basket contains 96 apples and another basket contains 60 oranges. If these fruits are to be packed into bags such that there is an equal number of apples in every bag and an equal number of oranges too in every bag and no fruits remain after they are packed into the bags, what is the maximum number of such bags that can be prepared?

4.6 Least Common Multiple (LCM)

Now let us consider what is meant by the least common multiple of several numbers and how it is found.

As an example, let us find the least common multiple of the numbers 2, 3 and 4.

- List the multiples of the given numbers.

Several multiples of the numbers 2, 3 and 4 are given in the following table.

Multiples of 2	2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26
Multiples of 3	3, 6, 9, 12, 15, 18, 21, 24
Multiples of 4	4, 8, 12, 16, 20, 24, 28

☛ Circle and write down the common multiples.

You will observe that the common multiples of the three numbers listed here are 12 and 24.

Further, if we continue to write the common multiples of 2, 3 and 4, we will obtain 12, 24, 36, 48, 60 etc

☛ The smallest of the common multiples of several numbers is called the **least common multiple** (LCM) of these numbers.

The smallest or the least of the common multiples 12, 24, 36, 48, 60, … of the numbers 2, 3 and 4 is 12.
Therefore, 12 is the least common multiple of 2, 3 and 4.

In other words, the smallest number which is divisible by 2, 3 and 4 is the least common multiple of 2, 3 and 4.

The least common multiple of several numbers is the smallest positive number which is divisible by all these numbers.

Note

- The HCF of several numbers is equal to or smaller than the smallest of these numbers
- The LCM of several numbers is equal to or larger than the largest of these numbers.
- The HCF of any two numbers is smaller than the LCM of the two numbers.

• Finding the LCM of several numbers by considering their prime factors

Let us see how the LCM of several numbers is found by considering their prime factors.

Let us find the LCM of 4, 12 and 18.

- Let us write each number as a product of its prime factors.

$$4 = 2 \times 2 = 2^2$$
$$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$$
$$18 = 2 \times 3 \times 3 = 2^1 \times 3^2$$

- Let us select the greatest power of each prime factor.

There are two distinct prime factors, namely, 2 and 3. When the factors of all three numbers are considered,

the power of 2 with the largest index = 2^2
the power of 3 with the largest index = 3^2.

- The LCM of the given numbers is the product of these greatest powers.

Therefore, the LCM of 4, 12 and 18 $= 2^2 \times 3^2$
$= 2 \times 2 \times 3 \times 3$
$= 36$

- **Finding the LCM by the method of division**

Let us find the LCM of 4, 12 and 18.

2	4,	12,	18
2	2,	6,	9
3	1,	3,	9
	1,	1,	3

- Write these numbers as shown.
- Since all these numbers are divisible by 2, divide each of them by 2 individually.
- We get 2, 6 and 9 as the result. No prime number divides all of them without remainder. However, 2 divides both 2 and 6 without remainder. Divide 2 and 6 by 2, and write the results below the respective numbers. Write 9 below 9.
- Since 3 and 9 are divisible by 3, the next smallest prime number, divide them by 3 and write the results below the respective numbers. Now observe that we cannot find at least two numbers which are divisible by the same number. Therefore, the division is stopped here.
- Multiply all divisors and all numbers left in the last row. The product gives the LCM of the given numbers.

Accordingly, the LCM of 4, 12 and 18 $= 2 \times 2 \times 3 \times 1 \times 1 \times 3 = 36$

Note
When using the method of division to find the LCM, keep dividing if there remain at least two numbers, divisible by another and obtain the LCM of the given numbers as above.

Let us find the LCM of 4, 3 and 5.

Here, we do not have at least two numbers which are divisible by a common number which is greater than 1.

Therefore, the LCM of 4,3 and 5 $= 4 \times 3 \times 5$

$= 60$

Example 1

Find the LCM of 8, 6 and 16.

Method I

Let us write each number as a product of its prime factors as follows.

8	$= 2 \times 2 \times 2$	$= 2^3$
6	$= 2 \times 3$	$= 2^1 \times 3^1$
16	$= 2 \times 2 \times 2 \times 2$	$= 2^4$

The distinct prime factors of these three numbers are 2 and 3. Here, the maximum number of times that 2 appears is four. The maximum number of times that 3 appears is one.

Therefore, the LCM of 8, 6 and 16 $= 2^4 \times 3$

$= 2 \times 2 \times 2 \times 2 \times 3$

$= 48$

Method II

$$\begin{array}{r|lll} 2 & 8, & 6, & 16 \\ \hline 2 & 4, & 3, & 8 \\ \hline 2 & 2, & 3, & 4 \\ \hline & 1, & 3, & 2 \end{array}$$

Since we cannot find another prime number by which at least two of the three numbers 1, 3, 2 are divisible, we stop the division here.

The LCM of 8, 6 and 16

$= 2 \times 2 \times 2 \times 1 \times 3 \times 2 = 48$

Example 2

(1) Two bells ring at intervals of 6 minutes and 8 minutes respectively. If they both ring together at 8.00 a.m., at what time will they ring together again?

* To find when both bells ring together again, it is necessary to find at what time intervals the bells ring together.

The first bell rings once every 6 minutes. 6, 12, 18, 24, …
The second bell rings once every 8 minutes. 8, 16, 24, …
Therefore, the two bells ring together again after 24 minutes.

The answer could be obtained by finding the LCM too.
Since the bells ring together at intervals of the LCM of 6 minutes and 8 minutes, to find out when the two bells ring together again, the LCM of these two numbers needs to be found.

$\therefore$ let us find the LCM of 6 and 8.

$$\begin{array}{r|l} 2 & 6,\ 8 \\ \hline & 3,\ 4 \end{array}$$

The LCM of 6 and 8 = $2 \times 3 \times 4 = 24$.
Therefore, the two bells ring together again after 24 minutes.
The bells ring together initially at 8.00 a.m.
Therefore, the bells ring together again at 8.24 a.m.

Exercise 4.7

(1) Find the LCM of each of the following triples of numbers.

(i) 18, 24, 36 (ii) 8, 14, 28 (iii) 20, 30, 40
(iv) 9, 12, 27 (v) 2, 3, 5 (vi) 36, 54, 24

(2) At a military function, three cannons are fired at intervals of 12 seconds. 16 seconds and 18 seconds respectively. If the three cannons are fired together initially, after how many seconds will they all be fired together again?

Miscellaneous Exercise

(1) Without dividing, determine whether the number 35 343 is divisible by 3, 4, 6 and 9.

(2) Fill in the blanks.

(i) The HCF of 2 and 3 is .... .
(ii) The LCM of 4 and 12 is .... .
(iii) The HCF of two prime numbers is .... .
(iv) The LCM of 2, 3 and 5 is .... .

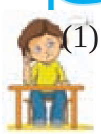
(3) Find the HCF and LCM of 12, 42 and 75.

(4) It is proposed to distribute books among 45 students in a class, such that each student receives no less than 5 books and no more than 10 books. Find all possible values that the total number of books that need to be bought can take, if all the students are to receive the same number of books and no books should be left over.

Summary

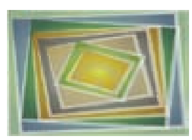
- Prime numbers among the factors of a number are called the prime factors of that number.
- Given two or more numbers, the largest of their common factors is called the highest common factor (HCF) of these numbers. Thus, the largest number which divides a list of numbers is their HCF.
- Given two or more numbers, the smallest of their common multiples is called the least common multiple (LCM) of these numbers. Thus, the smallest positive number which is divisible by a list of numbers is their LCM.

Ponder



(1) The length and breadth of a rectangular piece of cloth are 16 cm and 12 cm respectively. If this piece is to be cut without any waste into equal sized square pieces, what is the maximum possible side length of such a square piece?

(2) The length and the breadth of a floor tile are 16 cm and 12 cm respectively. What is the side length of the smallest square shaped floor area that can be tiled using such tiles, if no tile is to be cut?



(3) The circumference of the front and a back wheel of a tricycle are 96 cm and 84 cm respectively. What is the minimum distance the tricycle must move, for the front wheel and back wheels to complete full revolutions at the same instant?

(4) What is the smallest number which is greater than 19 that has a remainder of 19, when divided by 24, 60 and 36?

5 Indices

By studying this lesson you will be able to

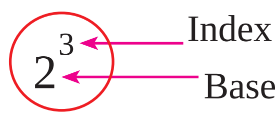
- write a number in index form, as a power having a prime number as the base,
- identify powers that have an algebraic symbol as the base,
- expand powers that have an algebraic symbol as the base and
- find the value of an algebraic expression by substituting positive integers for the unknowns.

Indices

Index notation is used to write a number which is multiplied repeatedly, in a concise way. Let us recall what has been learnt thus far about indices.

$2 \times 2 \times 2$ is written as 2^3 using indices.
That is, $2 \times 2 \times 2 = 2^3$.

In 2^3, 2 is defined as the base and 3 is defined as the index. 2^3 is read as "two to the power 3".



The value of 2^3 is 8. Therefore, the number 8 can be written as 2^3 in index notation.

When the index is a positive integer, it denotes how many times the number in the base is multiplied by itself.

Product	Number of times 3 is multiplied by itself	Index notation
3×3	2	3^2
$3 \times 3 \times 3$	3	3^3
$3 \times 3 \times 3 \times 3$	4	3^4
$3 \times 3 \times 3 \times 3 \times 3$	5	3^5
$3 \times 3 \times 3 \times 3 \times 3 \times 3$	6	3^6

You have learnt these facts in Grade 6. Do the following exercise to recall what you have learnt thus far about indices.

Review Exercise

(1) Expand each of the following as a product and find the value of the given expression.

(i) 3^2 (ii) 5^4 (iii) $2^2 \times 3$ (iv) $6^2 \times 5^2$

(2) Write down each of the following products using index notation.

(i) $4 \times 4 \times 4$ (ii) $7 \times 7 \times 7 \times 7$

(iii) $2 \times 2 \times 3 \times 3$ (iv) $3 \times 3 \times 5 \times 3 \times 5$

(3) Fill in the blanks in the following table.

Number	Index Notation	Base	Index	How the index notation is read
25	5^2	5	2	Five to the power two
343	....	7	....	....
....	....	....	....	Six to the power three

(4) Write the number 16

(i) using index notation with base 2.

(ii) using index notation with base 4.

5.1 Expressing a number in index notation with a prime number as the base

Let us write 8 in index notation with a prime number as the base.

Let us write 8 as a product of its prime factors.

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$8 = 2 \times 2 \times 2$

8 in index notation $= 2^3$

Now let us express the number 40 in index notation with prime numbers as the bases of the powers.

Let us write 40 as a product of prime numbers.

$$\begin{array}{r|l} 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$40 = 2 \times 2 \times 2 \times 5$

When this is written in index notation we obtain $40 = 2^3 \times 5$.

That is, 40 can be expressed as a product of powers with prime numbers as bases, in the form $40 = 2^3 \times 5$.

Do the following to express a number as a product of powers with prime numbers as bases.

- Start by dividing the number by the smallest prime number which divides it without remainder,
- Continue dividing the result by the prime numbers which divide it without remainder, in increasing order of the prime numbers, until the answer 1 is obtained.
- Write the number as a product of powers of these primes, where the index is the number of times division by that prime is done.

Example 1

Write down the number 36 as a product of powers with prime numbers as bases.

$$\begin{array}{r|r} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$36 = 2 \times 2 \times 3 \times 3$
$36 = 2^2 \times 3^2$

Example 2

Write down the number 100 as a product of powers with prime numbers as bases.

$$\begin{array}{r|r} 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$100 = 2 \times 2 \times 5 \times 5$
$100 = 2^2 \times 5^2$

Exercise 5.1

(1) (i) Write 25 in index notation with 5 as the base.
(ii) Write 64 in index notation with 2 as the base.
(iii) Write 81 in index notation with 3 as the base.
(iv) Write 49 in index notation with 7 as the base.

(2) Write each of the following numbers as a product of powers with prime numbers as bases.
(i) 18 (ii) 24 (iii) 45 (iv) 63 (v) 72

5.2 Powers with an algebraic symbol as the base

We have learnt about powers with a number as the base. Let us now consider instances when the base is an algebraic symbol.

$2 \times 2 \times 2 = 2^3$
$5 \times 5 \times 5 = 5^3$

We can in a similar manner write $x \times x \times x = x^3$.

The base of x^3 is x and the index is 3.

Index
x^3
Base

Further,

$a \times a$ and $m \times m \times m \times m$ can be expressed as powers as
$a \times a = a^2$ and $m \times m \times m \times m = m^4$ with an algebraic symbol as the base.

$2^1 = 2$. Accordingly, a can be written as $a = a^1$ in index notation.
The product of 2 and 3 is written as 2×3.
The product of a and b can be written as $a \times b$.
$a \times b$ can be expressed as ab or ba.
Accordingly $3 \times a \times b$ can be expressed as $3ab$.

Further, $m \times m \times m \times n \times n = m^3 \times n^2$.

$m^3 \times n^2$ which is also equal to $n^2 \times m^3$, can be expressed as m^3n^2 or as n^2m^3.

When two powers are connected with a multiplication sign, if the bases of both the powers are not numerical values, then it is not necessary to include the multiplication sign.

Example 1

Write down each of the following expressions using index notation.

(i) $p \times p \times p$
(ii) $x \times x \times y \times y \times y$
(iii) $2 \times 2 \times a \times a \times a$
(iv) $m \times 3 \times m \times 3 \times 3$

(i) $p \times p \times p = p^3$
(ii) $x \times x \times y \times y \times y = x^2 \times y^3 = x^2 y^3$
(iii) $2 \times 2 \times a \times a \times a = 2^2 \times a^3 = 2^2 a^3$
(iv) $m \times 3 \times m \times 3 \times 3 = 3^3 \times m^2 = 3^3 m^2$

Example 2

Expand and write each of the following expressions as a product.

(i) m^3
(ii) $p^2 q^3$
(iii) $5^2 x^3$

(i) $m^3 = m \times m \times m$
(ii) $p^2 q^3 = p \times p \times q \times q \times q$
(iii) $5^2 x^3 = 5 \times 5 \times x \times x \times x$

Exercise 5.2

(1) Write down each of the following expressions using index notation.

(i) $x \times x \times x \times x$ (ii) $a \times a \times a$ (iii) $m \times m \times m \times n \times n \times n$

(iv) $7 \times 7 \times 7 \times p \times p$ (v) $y \times y \times y \times y \times 7 \times 7 \times 7$

(2) Expand and write each of the following expressions as a product.

(i) a^2 (ii) $2p^2$ (iii) $2^3 m^2$ (iv) $3^2 x^3$ (v) $x^3 y^3$

5.3 Finding the value of a power by substitution

Let us consider expressions in index notation with bases which are unknowns. By substituting values for the unknown bases, the value of an expression in index notation can be found. In this lesson, only positive integers are substituted.

Let us find the value of the expression x^3 when $x = 2$.

Method I	Method II
By substituting the value 2 for x we obtain, $x^3 = 2^3$ $= 2 \times 2 \times 2$ $= 8$	$x^3 = x \times x \times x$ By substituting the value 2 for x we obtain, $x^3 = 2 \times 2 \times 2$ $x^3 = 8$

Example 1

Find the value of each of the following expressions when $x = 5$.

(i) x^3

Method I

$x^3 = 5^3$
$= 5 \times 5 \times 5$
$= 125$

Method II

$x^3 = x \times x \times x$
$= 5 \times 5 \times 5$
$= 125$

(ii) $3x$

$3x = 3 \times x$
$= 3 \times 5$
$= 15$

Example 2

Find the value of each of the following expressions when $a = 3$ and $b = 5$.

(i) $a^2 b$ (ii) $2a^3 b^2$

(i) $a^2 b$

$a^2 b = a \times a \times b$

Substituting $a = 3$ and $b = 5$

we obtain,

$a^2 b = 3 \times 3 \times 5$

$= 45$

(ii) $2a^3 b^2$

$2a^3 b^2 = 2 \times a \times a \times a \times b \times b$

Substituting $a = 3$ and $b = 5$

we obtain,

$2a^3 b^2 = 2 \times 3 \times 3 \times 3 \times 5 \times 5$

$= 1350$

Exercise 5.3

(1) Find the value of each of the following expressions by substituting $x = 3$.

(i) x^4 (ii) $3x^2$ (iii) $5x^3$

(2) Find the value of each of the following expressions by substituting $a = 3$.

(i) $2a^2$ (ii) 2^2a^2 (iii) $7a^2$

(3) Find the value of each of the following expressions by substituting $x = 1$ and $y = 7$.

(i) $x^2 y^3$ (ii) $2x^3 y$ (iii) $3xy^2$

(4) Find the value of each of the following expressions by substituting $a = 2$ and $b = 7$.

(i) $a^2 b$ (ii) ab^2 (iii) a^3b^2 (iv) $3a^2b^2$

Summary

- An expression of an unknown term multiplied repeatedly can be expressed as a power with the unknown term as the base and the number of times the term is multiplied as the index.

a to the power three $\rightarrow a^3$ Index Base

- When two powers are connected with a multiplication sign, if the bases of both the powers are not numerical values, then it is not necessary to include the multiplication sign.
- A value can be obtained for an expression in index notation with an unknown base, by substituting a number for the unknown term.

6 Time

By studying this lesson you will be able to
- identify months, years, decades, centuries and millenniums as units of time
- identify a leap year,
- identify the relationships between units of time, and
- add and subtract units of time.

6.1 Units of time

You have already learnt that the units seconds, minutes , hours and days are used to measure time.

You have also learnt to find the time it takes to do different daily activities.

Now, let us learn more on the units of measuring time - months, years, decades, centuries and millenniums.

• Months and years

If we want to calculate the time taken for an event which commences on a particular date and ends on another date, in terms of days, weeks or months, we can do so by looking at a calendar.

A calendar is made up of the units days, weeks and months. You will see that there are 12 months in a calendar.

The calendar of year 2015 is shown below. The table shows the number of days in each month.

2015

The months having 31 days	The months having 30 days	The months having 28 days
January	April	February
March	June	
May	September	
July	November	
August		
October		
December		

The calendar of a particular year provides information on a period of a year, starting from the first of January and ending on the thirty first of December of that year.

According to the year 2015 calendar, the total number of days in the year is 365. There are 365 days in a year which is not a leap year. We will be studying about leap years later.

- The day 2015-08-01 means, the time period from 00:00 on 2015-08-01 to 24:00 on 2015-08-01.
- The time at which a particular day ends ,is the time at which the next day starts. So the time 24:00 on 2015-08-01 is the same as the time 00:00 on 2015-08-02.
- The year 2015 means, the time period from 00:00 on 2015-01-01 to 24:00 on 2015-12-31.

Note :
The international convention for measuring years is by considering the year of the birth of Jesus.

BC and AD are commonly used to count years in time. Jesus Christ's birth is used as the starting point to count years that existed before (BC) and after (AD) he was born.

Decades

A time period of ten years is considered as a decade. Let us consider 1948.

The first year in the decade that contains the year 1948 is 1941, and the last year in that decade is 1950.

The time period from AD 1 to AD 10 is called the first decade.
The time period from AD 11 to AD 20 is called the second decade.
The time period from AD 1811 to AD 1820 is called the hundred and eighty second decade.
The time period from AD 1951 to AD 1960 is called the hundred and ninety sixth decade.
The time period from AD 2011 to AD 2020 is called the two hundred and second decade.

That is, the time period from time 00:00 on 1941-01-01 to time 24:00 on 1950-12-31 is a decade. This decade is identified as the 195^{th} decade.

Centuries

A time period of a hundred years is called a century.

AD 1 to AD 100 is the first century.
AD 101 to AD 0200 is the second century.
AD 1801 to AD 1900 is the nineteenth century.
AD 1901 to AD 2000 is the twentieth century.
AD 2001 to AD 2100 is the twenty first century.

The time period from 00:00 on 2001-01-01 to 24:00 on 2100-12-31 is the twenty first century.

A Millennium

A time period of a 1000 years is known as a millennium. According to the calendar, at this moment we are living in the third millennium.

The time period from AD 1 to 1000 is the first millennium.
The time period from AD 1001 to 2000 is the second millennium.

Example 1

(i) To which millennium does AD 1505 belong? Second millennium

(ii) To which century does AD 1505 belong ? 16th century

(iii) To which decade does AD 1505 belong? Hundred and fifty first decade.

Exercise 6.1

(1) Write down the decade to which each one of the following years belongs.

(i) AD 1856 (ii) AD 1912 (iii) AD 1978 (iv) AD 2004

(2) Write the first date and the last date of the 22nd century.

(3) Write down the century to which each one of the following years belongs.

(i) AD 1796 (ii) AD 1815 (iii) AD 1956 (iv) AD 2024

6.2 Leap year

The calendar of 2016 is given below. Consider the number of days in each month . How does this differ from the calendar of 2015 ?

The months having 31 days	The months having 30 days	The months having 29 days
January	April	February
March	June	
May	September	
July	November	
August		
October		
December		

There are 29 days in the month of February. So the total number of days in 2016 is 366.

Any year in which there are 29 days in the month of February has 366 days in total. Such a year is defined as a leap year.

If a number that denotes a year is divisible by 4 but is not a multiple of 100, then that year is a leap year. However years which are denoted by numbers that are multiples of 100 become leap years only if they divisible by 400.

∴

Example 1

Is the year 2000 a leap year?

Since $2000 = 100 \times 20$, 2000 is a multiple of 100.
Since $2000 \div 400 = 5$, 2000 is divisible by 400.
So year 2000 is a leap year.

Example 2

Is the year 1900 a leap year?
1900 is a multiple of 100.
1900 is not divisible by 400.
∴ 1900 is not a leap year.

Example 3

Is the year 2008 a leap year?
2008 is not a multiple of 100.
$2008 \div 4 = 502$, 2008 is divisible by four .
∴ 2008 is a leap year.

Example 4

Is 2010 a leap year?
2010 is not a multiple of 100.

The number that is formed by the last two **digits** of 2010, that is 10, is not divisible by 4. Therefore by the rules of divisibility, 2010 is also not divisible by 4.

Hence, 2010 is not a multiple of 4. Therefore 2010 is not a leap year.

Note : Any year which is not a multiple of 4 is not a leap year.

Further units of time

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day

There are months consisting of 28 days, 29 days, 30 days and 31 days.

However a time period of 30 days is considered as a month.

12 months = 1 year
365 days = 1 year
366 days = 1 leap year

A time period given in years can be represented in days, by multiplying it by 365.

A time period given in years can be represented in months, by multiplying it by 12.

Note :
We consider 30 days as a month. However, because a year consists of 12 months, you should not think that the number of days in a year is 360 (12 × 30 days). A year consists of 365 days.

Example 1

(i) Indicate 280 days in months and days.

$$\begin{array}{r|l} & 9 \\ \hline 30 & 280 \\ & \underline{270} \\ & 10 \end{array}$$

Therefore, 280 days is 9 months and 10 days.

Example 2

(i) Indicate 3 years in months.
(ii) Indicate 3 years in days.

(i) 3 years $= 3 \times 12$ months
$= 36$ months

(ii) 3 years $= 3 \times 365$ days
$= 1095$ days

6.2 Exercise

(1) Choose the leap years from the years given below.

(i) AD 1896 (ii) AD 1958 (iii) AD 1960
(iv) AD 1400 (v) AD 1600 (vi) AD 2016

(2) (a) Indicate the days given below, in months and days.

(i) 225 days (ii) 100 days (iii) 180 days

(b) How many months are there in 5 years? How many days are there in 5 years?

(3) A bus which makes 4 trips a day, runs continuously for 6 months daily. Find the total number of trips it makes during this period.

(4) A patient has to take 3 tablets per day for a period of 2 months. How many tablets are required for this purpose?

(5) A person exercises for 1 hour every day.

(i) How many hours does he spend exercising during a year which is not a leap year?

(ii) Indicate this time in days.

(6) A person puts a minimum of 5 Rupees in a till every day. Find the least amount of money he would collect during each time period below.

(i) 6 months (ii) A leap year

6.3 Calculations related to time

A certain school was in session as follows during a certain year.

The first term consisted of 3 months and 6 days, the second term consisted of 3 months and 8 days, and the third term consisted of 3 months and 3 days.

	Months	Days
	3	6
	3	8
+	3	3
	9	17

Let us express the time period that the school was in session that year, in months and days.

For this we need to add the above time periods to find the total time period.

So the school was in session for 9 months and 17 days.

Example 1

A teacher served for 5 years 6 months and 23 days in a school in the Eastern province and for 6 years 8 months and 15 days in a school in the Central province. He served the rest of his career in a school in the Southern province.

(i) Find in total how long he served in the Eastern and Central provinces.
(ii) If he served for 28 years, 2 months and 2 days in total, then find how long he served in the school in the Southern province.

(i)

	Years	Months	Days
	5	6	23
+	6	8	15
			8

Let us add the days in the "days column"
23 days + 15 days = 38 days
38 days = 1 month + 8 days
Let us write the 8 days in the "days column". Let us carry the 1 month to the "months column".

	Years	Months	Days
	5	6	23
+	6	8	15
	12	3	8

1 month + 6 months + 8 months = 15 months = 1 year and 3 months
Let us write the 3 months in the "months column".
Let us carry the year to the "years column".

1 year + 5 years + 6 years =12 years.

The total service of the teacher in the Eastern and Central provinces is 12 years, 3 months and 8 days.

(ii)

	Years	Months	Days
	28	2	2
–	12	3	8
			24

Let us subtract the days in the "days column".
Since 2<8 let us take a period of one month, that is 30 days from the "months column" and add it to the "days column".
Then, 30 days + 2 days = 32 days.
32 days - 8 days = 24 days.
Let us write the 24 days in the "days column".

	Years	Months	Days
	28	2	2
–	12	3	8
	15	10	24

Now, in the months column we have to subtract 3 months from the remaining 1month. Since this cannot be done, let us carry a period of 1 year, that is 12 months, from the "years column" to the "months column". Then,
12 months + 1 month = 13 months
13 months – 3 months = 10 months
Let us write the 10 months in the " months column".

When 12 years are deducted from the remaining 27 years in the "years column" we get 15 years.

So the amount of time the teacher spent in the school in the Southern province is 15 years, 10 months and 24 days.

Example 2

Sunitha's date of birth is 2008-05-06.

(i) What is her age on 2016-08-24?

(ii) Nimal is younger to her by 3 years, 6 months and 3 days. Find Nimal's date of birth.

(i) The date on which we want to know the age = 2016-08-24

Sunitha's date of birth = 2008-05-06

Let us find Sunitha's age on 2016-08-24.

	Years	Months	Days
	2016	8	24
−	2008	5	6
	8	3	18

Sunitha's age is 8 years, 3 months and 18 days.

(ii) Nimal's date of birth is the ninth of November, 2011.

	Years	Months	Days
	2008	5	6
+	3	6	3
	2011	11	9

Exercises 6.3

(1) Do the following additions.

(i)

	Months	Days
	8	18
+	2	11

(ii)

	Months	Days
	8	22
+	2	16

(iii)

	Years	Months	Days
	12	6	21
+	3	2	19

(iv)

	Years	Months	Days
	8	9	19
+	2	6	23

(2) Do the following subtractions.

(i)

	Months	Days
	6	23
−	3	15

(ii)

	Months	Days
	6	18
−	2	24

(iii)

	Years	Months	Days
	3	6	15
−	2	4	18

(iv)

	Years	Months	Days
	2	8	12
−	1	2	15

(3) Dileepa's date of birth is 2003-09-07.
Sithumini's date of birth is 2000-02-04.

(i) Find how old Dileepa and Sithumini are today.
(ii) Find how much older Sithumini is to Dileepa,
(a) using their ages,
(b) using their dates of birth.

(4) Below are the service periods of two teachers in a certain school.

	Date he started work in the school	**The date he was transferred from this school**
Mr. Iqbal	2001 - 07 - 13	2015 - 11 - 22
Mr. Subhairudeen	1997 - 03 - 20	2012 - 01 - 10

(i) Find the period of service of each teacher. Who has served longer in this school?
(ii) How many more years has the teacher has served than the other one ?

(5) Shashika's date of birth is 2014-08-13. Aheli is 1 year, 8 months and 25 days older to her. What is Aheli's date of birth?

(6) A school was opened on 1928-03-26.
(i) When is the school's centennial anniversary?
(ii) How many days are there to the centennial anniversary date from today?

(7) Amila participated in Agricultural training progammes in Japan and China. He stayed in Japan from 2012-02-13 to 2014-07-27 and in China from 2014-12-17 to 2015-10-05. Find the total time he spent in Japan and China.

Miscellaneous Exercise

(1) A person borrows a certain amount of money. He has to pay the debt in equal installments once every month, for 10 years. The first installment was paid on 2016-01-01. Find the date on which he has to pay the final installment.

(2) Below are the age limits for participants in an inter-house sportsmeet of a certain school.

Under 11 games – Age should be less than 11 years on 2016-03-31.

Under 13 games – Age should be less than 13 years and greater than or equal to 11 years on 2016-03-31.

(3) Under 15 games – Age should be less than 15 years and greater than or equal to 13 years on 2016-03-31.

Under 17 games – Age should be less than 17 years and greater than or equal to 15 years on 2016-03-31.

The dates of birth of several students are given below.

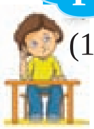
Name	Birthday
Vanthula	2005-12-08
Hashan	2002-05-17
Hasintha	2000-01-16

Find which age group each student qualifies to participate in.

Summary

- A time period of 10 years is defined as a decade.
- A time period of 100 years is defined as a century.
- A time period of 1000 years is defined as a millennium.
- If a number that denotes a year is divisible by 4 but is not a multiple of 100, then that year is a leap year. However years which are denoted by numbers that are multiples of 100 become leap years only if they divisible by 400.

Ponder

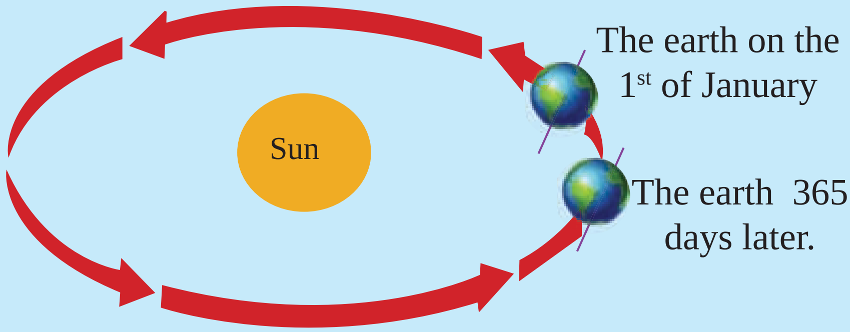


(1) A person was born on 2002-09-23 at 9.32 a.m. Find for how long he has lived in years, days, hours and minutes when it is 12 noon of 2015-06-05.

(2) A certain person lived for 20591 days. Find his age in years, months and days at the time he passed away.

Extra Knowledge –More on leap years

• Why Do We Have Leap Years?



In any year which is not a leap year there are 365 days.
A year is defined as the time it takes for the Earth to orbit around the sun once.

However, the exact time it takes for the earth to orbit around the sun is 365 days 5 hours 48 minutes and 46 seconds. This is about $365\frac{97}{400}$ days. So when we say a year has 365 days, we have neglected a time period of 5 hours, 48 minutes and 46 seconds (**which is little less than 1/4 day**). Four of these periods added together is approximately one day. We add this as an extra day to the calendar once every four years. It is added to the month of February. This is how we get a leap year.

A leap year has an extra day. Due to the decision to add an extra day to the calendar once every four years, 3 additional days get included in each 400 year period.

Although an extra day is added once every four years, only 23 hours 15 minutes and 4 seconds should actually be added once every four years. Therefore, due to the decision of adding an extra day once every four years, there are approximately 3 additional days that are included in each period of 400 years.

Therefore three days need to be removed from every 400 year period.

To do this, an extra day is not added to the month of February for the first three century years. (A century year is a year ending in 00)

Non-century years are leap years if they are multiples of four.

7 Parallel Straight Lines

By studying this lesson you will be able to
- identify parallel straight lines,
- identify that the gap between a pair of parallel straight lines is the perpendicular distance, that is, the shortest distance between the two lines,
- examine whether a given pair of straight lines is parallel or not by using a straight edge and a set square,
- draw parallel lines using a straight edge and a set square, and
- draw rectilinear plane figures containing parallel lines using a straight edge and a set square.

7.1 Straight line segment

Activity 1

(1) Draw a straight line using a straight edge. Name this straight line l.

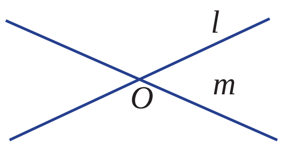
(2) Mark the two points A and B on the straight line l as shown in the figure.

The portion AB of the straight line l is defined as the straight line segment AB. The two points A and B are defined as **the two end points of the straight line segment** AB.

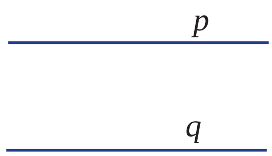
The convention is to use capital letters of the English alphabet to name straight line segments.

7.2 Parallel straight lines

Examine the two pairs of straight lines given below which are drawn on the same plane.



The straight lines l and m intersect each other at O.



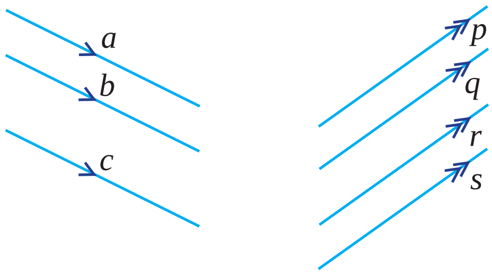
The straight lines p and q do not intersect each other.

Two straight lines which do not intersect each other are called **parallel straight lines**.

Accordingly, the two straight lines p and q are parallel, while the two straight lines l and m are not parallel.

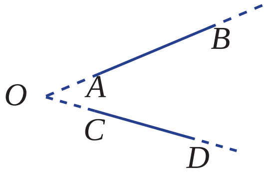
When several straight lines do not intersect each other, they are defined as straight lines which are parallel to each other.

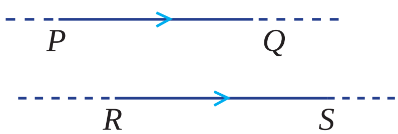
To indicate that several lines are parallel to each other, arrowheads are drawn on the straight lines in the same direction and sense, as shown in the figure.



Accordingly, in the above figure, a, b and c are parallel to each other and p, q, r and s are parallel to each other.

Let us check whether each of the following pairs of straight line segments are parallel to each other or not.





The two straight lines on which the straight line segments *AB* and *CD* lie, intersect at *O*. However the two straight lines on which the straight line segments *PQ* and *RS* lie, do not intersect.

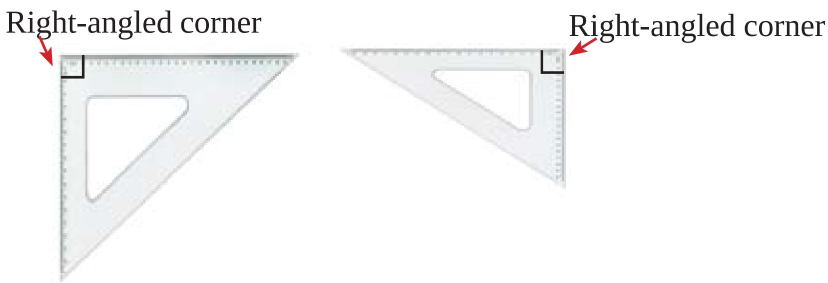
Accordingly, *PQ* and *RS* are parallel straight line segments while *AB* and *CD* are not.

We indicate the fact that *PQ* and *RS* are parallel straight line segments using the notation “*PQ* // *RS*”.

7.3 Perpendicular distance

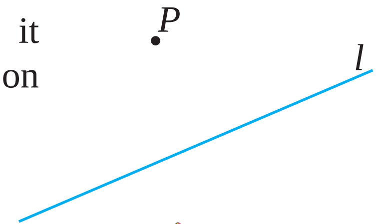
• The perpendicular distance from a point to a straight line

The following is a figure of set squares. Let us consider how the perpendicular distance from a point to a straight line is found using a set square.

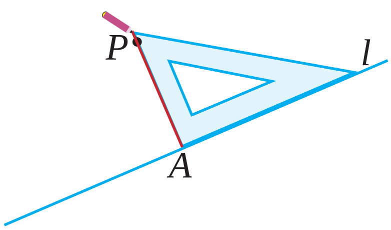


Activity 2

(1) Draw a straight line and name it *l*. Mark a point *P* which is not on *l*.

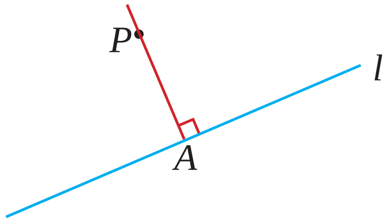


(2) As shown in the figure, place the set square such that one edge which forms the right angled corner lies on *l* and the other edge passes through the point *P*.



(3) Mark the point *A* on *l* as indicated and join *AP*.

The angle marked at A is a right angle. **We say that the straight line segment AP is perpendicular to l.**

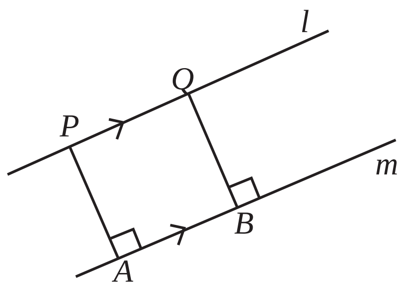


(3) Observe that the point on l which is closest to P is A. Measure AP.

The length of the straight line segment AP is defined as the **perpendicular distance** from the point P to the straight line l. The length of AP is the **shortest distance** from the point P to l.

- **The perpendicular distance between two parallel straight line**

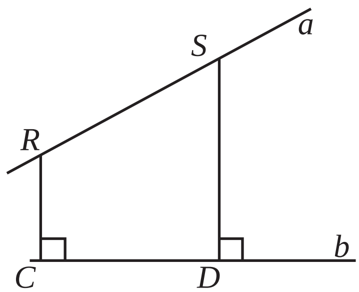
The perpendicular distances from the two points P and Q that lie on the line l to the straight line m are equal to each other. That is, $PA = QB$.



$\therefore l$ and m are two parallel straight lines.

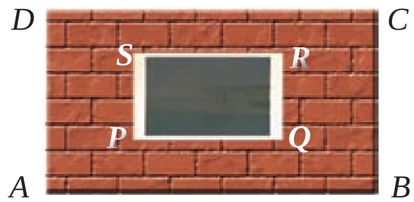
However, the perpendicular distances from the two points R and S on a to the straight line b are unequal. That is, $RC \neq SD$.

$\therefore$The straight lines a and b are not parallel to each other.



- The shortest distance from every point on a straight line to a parallel straight line is a constant. This constant distance is defined as the **perpendicular distance** between the two parallel straight lines.
  This perpendicular distance is also defined as the **gap** between the two parallel straight lines.
- Straight lines which lie on the same plane and which are a constant distance from each other are parallel to each other.

The figure given below depicts a wall of a room and a window in the wall. Since the wall is rectangular in shape, the opposite edges are parallel.



- That is, the horizontal edges which are represented by the straight line segments *AB* and *DC* are parallel to each other.
- Similarly, the vertical edges which are represented by the straight line segments *AD* and *BC* are parallel to each other.
- The straight line segments *PQ* and *SR*, represent the horizontal edges of the window. They are parallel to each other.
- The straight line segments *PS* and *QR*, represent the vertical edges of the window. They are parallel to each other.

There are several locations in the environment where such parallel edges can be observed.

- The horizontal panels of a ladder
- The beams of a roof
- The straight line segments of a 100 m running track

are some examples.

Exercise 7.1

(1) Write down the names of two objects that can be observed in the classroom that have parallel edges.

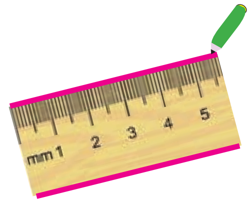
(2) Write down the names of two objects in your day to day environment that have parallel edges.

(3) Name four locations where parallel lines can be observed in architectural designs.

(4) Describe several arrangements and tasks which involve parallel straight lines.

7.4 Drawing parallel lines using a straight edge and a set square

As shown in the figure, place the ruler on a page of your exercise book and draw two straight lines along the edges of the ruler. Now you have obtained a pair of parallel straight lines.



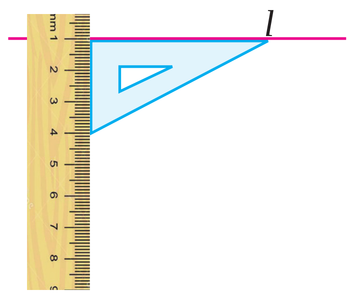
• Drawing a straight line parallel to a given straight line using a straight edge and a set square

Activity 3

(1) Draw a straight line using a straight edge and name it l.

(2) Place the set square such that one edge which forms the right angled corner lies on the straight line l.

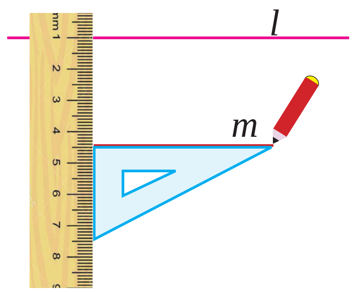
As shown in the figure, place the straight edge such that it touches the other edge which forms the right angled corner of the set square.



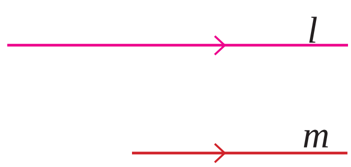
(3) Keeping the straight edge fixed, move the set square along the straight edge.

(4) Stop moving the set square and draw a straight line along the edge which forms the right angled corner and is not touching the straight edge.

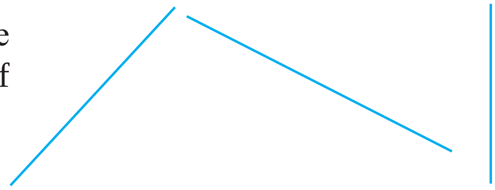
(5) Name this straight line m.



Now you have obtained a straight line m which is parallel to the straight line l.



➢ Copy the straight lines in the figure and draw a line parallel to each of them.

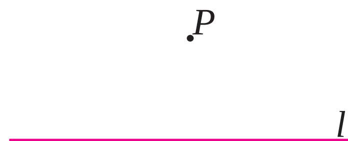


Only one line can be drawn parallel to a given line on a plane, through a point on the plane which does not lie on the given line.

- **Drawing a straight line parallel to a given straight line, through a point which is not on the straight line, using a straight edge and a set square**

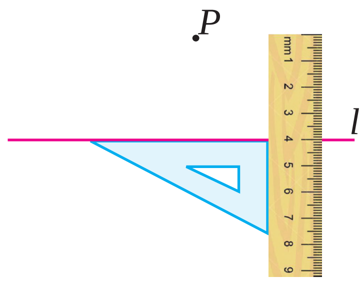
Activity 4

(1) As indicated in the figure, name a point P that does not lie on the straight line l.



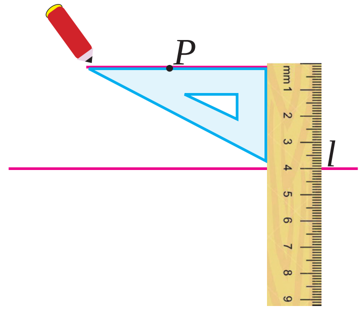
(2) Place the set square such that one edge which forms the right angled corner lies on the straight line l.

As shown in the figure, place the straight edge such that it touches the other edge which forms the right angled corner of the set square.



(3) Keeping the straight edge fixed, move the set square along the straight edge.

(4) When the edge of the set square which was lying on the straight line l touches the point P, draw a straight line along it.



Now you have obtained a straight line through the point P, which is parallel to the straight line l.

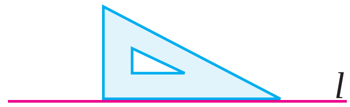
• Drawing a line parallel to a straight line at a given distance from the straight line, using a ruler and a set square

Activity 5

Let us draw a straight line which is parallel to the straight line *l* through a point at a distance of 2.5 cm above it.

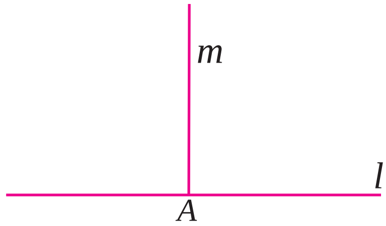
(1) Draw a straight line *l* as shown in the figure.

(2) Place the set square such that one edge which forms the right angled corner lies on the straight line *l*.



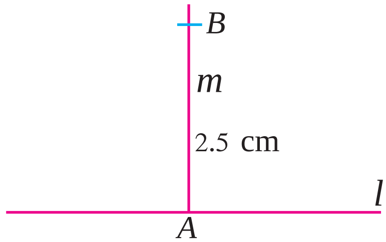
(3) Draw a straight line along the edge which forms the right angled corner and does not lie on the straight line *l*.

Name this straight line *m*.

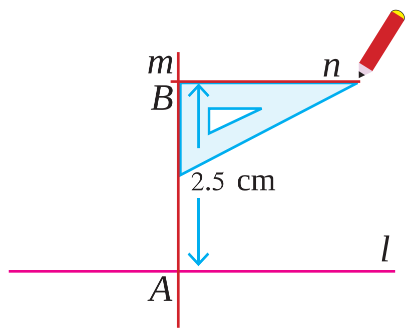


(4) Name the point at which the straight line *m* meets the straight line *l* as *A*.

(5) Mark the point *B* on the straight line *m*, at a distance of 2.5 cm from *A*.



(6) Place the set square such that the right angled corner coincides with *B* and one of the edges which forms the right angled corner lies on *m*, and draw the line *n* along the other edge which forms the right angled corner.



Now you have obtained a straight line *n* which is parallel to the straight line *l* and which lies at a distance of 2.5 cm from *l*.

(7) Draw in a similar manner, the straight line which is parallel to the straight line *l* and which lies 2.5 cm below *l*.

Exercise 7.2

(1) (i) Draw a straight line segment of length 6 cm and name it AB.

(ii) Mark a point P which does not lie on the straight line segment.

(iii) Draw a straight line passing through P parallel to AB, using a ruler and a set square.

(iv) Find the gap between the two straight lines by using a straight edge and a set square.

(2) (i) Draw a straight line segment. Name it PQ.

(ii) Mark a point A below PQ such that the perpendicular distance from A to PQ is 4.8 cm.

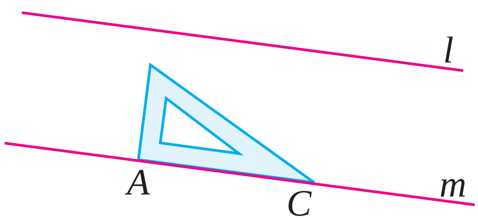
(iii) Draw a straight line segment which passes through A and is parallel to PQ.

7.5 Examining whether two straight lines are parallel

To determine whether two straight lines in the same plane are parallel or not, it is necessary to check whether the perpendicular distances from any two points on one line to the other line are equal or not.

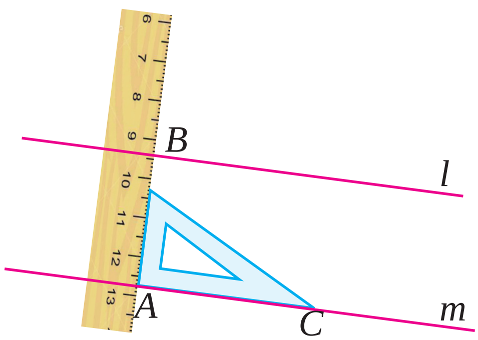
Activity 6

Let us examine whether the two straight lines l and m are parallel.

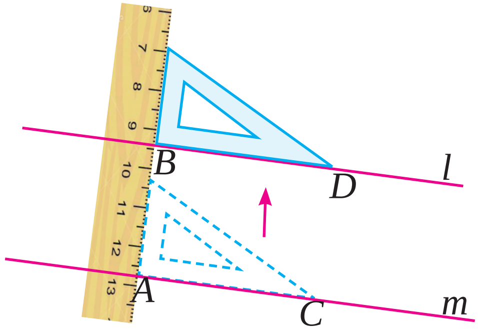


(1) As shown in the figure, place the set square such that one edge which forms the right angled corner lies on the straight line m.

(2) Place the straight edge such that it touches the other edge which forms the right angled corner of the set square as indicated in the figure. Name the point at which the straight edge meets the line l as B.



(3) Keeping the straight edge fixed, move the set square along the straight edge as shown in the figure, until the right angled corner coincides with the point B on the straight line l.



(4) Check whether the edge of the right angled corner which was initially on the straight line m, now coincides with the straight line l.

If it coincides, then the perpendicular distances from the two points B and D to the straight line m will be equal, and hence l and m are two parallel straight lines.

If it does not coincide, then the straight lines l and m are not parallel to each other.

7.6 Drawing rectilinear plane figures using a set square and a straight edge

Activity 7

(1) Draw a rectangle of length equal to the length of 6 squares and breadth equal to the length of 4 squares on your square ruled exercise book.

(2) Establish that the distance between the two longer sides of the rectangle is a constant value by counting squares. Confirm this by measuring the distance between the two longer sides using a ruler.

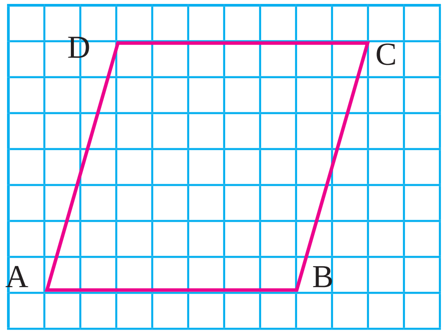
- If the distance is a constant value, then the straight line segments which represent the two longer sides of the rectangle are parallel to each other.
- It can be seen similarly that the two straight line segments drawn to represent the shorter sides of the rectangle are also parallel to each other.

Activity 8

(1) Draw the straight line segments AB and DC on a square ruled paper such that their lengths are equal to the length of 7 squares.

(2) Complete the figure $ABCD$ by drawing the straight line segments AD and BC.

(3) Using a set square and a straight edge, show that AD and BC are parallel to each other and find the gap between them.



Activity 9

(1) Draw a straight line segment and mark the points A and B on it such that $AB = 6$ cm.

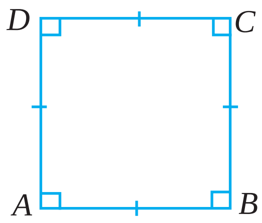
(2) Using a set square, draw two straight lines through the points A and B, perpendicular to the given line.

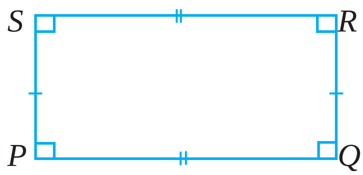
(3) Mark the points C and D such that $AD = 6$ cm and $BC = 6$ cm.

(4) Complete the figure $ABCD$ using a straight edge. What is the name given to a quadrilateral of the shape $ABCD$?

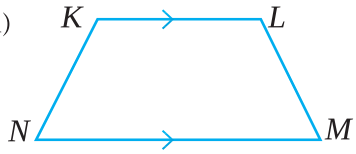
Exercise 7.3

(1) Draw each of the following figures using a straight edge and a set square.

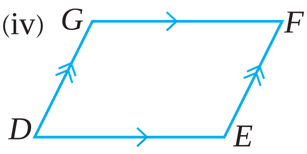
(i) 

(ii) 

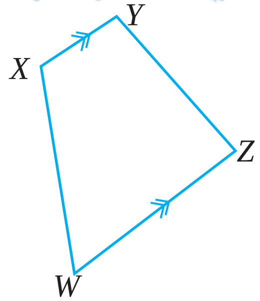
(iii)



(iv)



(v)



(2) Write down for each of the above figures whether each pair of opposite sides is parallel or not.

(3) Using a straight edge and a set square,
  (i) draw a square of side length 5 cm.
  (ii) draw a rectangle of length 8 cm and breadth 5 cm.

(4) (i) Draw a straight line segment AB such that $AB = 6$ cm.
  (ii) Draw the straight line segment BC such that if forms an obtuse angle with AB at B.
  (iii) Draw a straight line through C parallel to AB in the direction of A.
  (iv) Mark the point D on this straight line such that $CD = 6$ cm. Join AD to obtain the parallelogram $ABCD$.

Summary

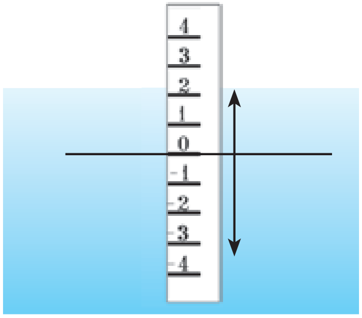
- Two straight lines in a plane which do not intersect each other are called parallel straight lines.
- Two straight lines in a plane which are at a constant distance from each other are parallel to each other.
- The gap between two parallel straight lines is a constant.

8 Directed Numbers

By studying this lesson you will be able to

- identify what directed numbers are,
- add integers using the number line, and
- add directed numbers without using the number line.

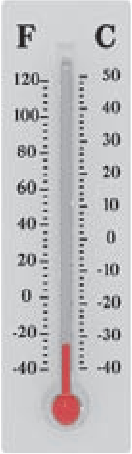
8.1 Identifying directed numbers



The figure given here represents an indicator that is used to measure the water level of a reservoir from which water is distributed to a certain city.

The usual water level of the reservoir has been marked as 0 (zero), and the indicator has been calibrated such that the gaps between the numbers above the 0 limit and below the 0 limit are equal.

Thereby it can be observed whether the water level of the reservoir is above or below 0 (the usual level). Here, by calibrating the indicator in opposite directions, a correct perception of the water level of the reservoir is obtained.



Similarly, thermometers that are used to measure the temperature of the environment are calibrated in opposite directions from 0^o C, to indicate temperatures that are greater than 0^o C and temperatures that are less than 0^o C.

The thermometer in the figure has been calibrated with the values 10, 20, 30, … in one direction to indicate the temperatures that are greater than 0^{o} C, and with the values –10, –20, –30, … in the opposite direction to indicate the temperatures that are less than 0^{o} C.

Let us now consider the number line given below.

The positive whole numbers marked to the right of the position indicating zero on the number line are defined as positive integers and the negative whole numbers marked to the left of the position indicating zero are defined as negative whole numbers.

{…, –3, –2, –1, 0, 1, 2, 3, …} is the set consisting of all the integers.

Any positive number can be marked on the above number line to the right of the position indicating and any negative number can be marked to the left of the position indicating 0, taking into consideration the magnitude of the number.

All the numbers that are written with a positive or negative sign to indicate not only their magnitude but also one of two directions which are opposite to each other are defined as **directed numbers**.

Accordingly, numbers such as $+4$, $+\frac{3}{4}$, $+5.7$, -10, $-\frac{1}{3}$ and -3.2 are directed numbers. $+4$ is read as positive four and $-\frac{1}{3}$ is read as negative one third.

Note

- When a sign is not written in front of a number, it is considered as a positive number.

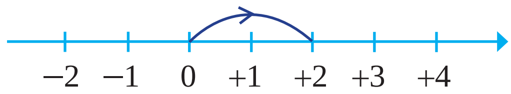
8.2 Adding directed numbers which are integers by using the number line

Let us consider adding directed numbers which are positive integers by using the number line.

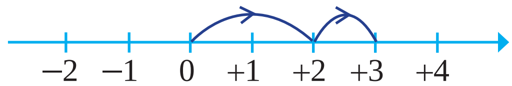
- **The sum of two positive integers**

Let us find the value of $(+2) + (+1)$ using the number line.

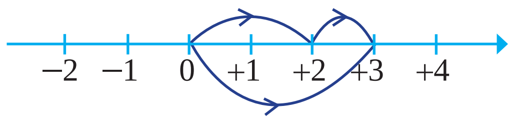
First, starting from 0, let us go two units towards the right along the number line.



Next, from this point, let us go one unit towards the right along the number line.



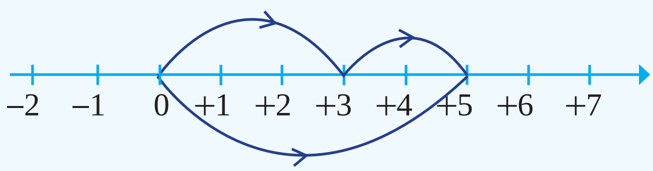
The directed number denoted by the position at which we finally stop is the answer.



$(+2) + (+1) = (+3)$

Example 1

Find the value of $(+3) + (+2)$ using the number line.



The final position is five units to the right of 0.

$\therefore\ (+3) + (+2) = (+5)$

Exercise 8.1

Find each of the following sums using the number line.

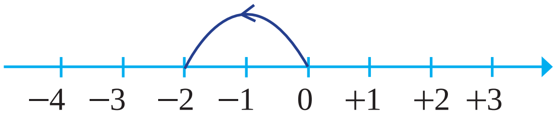
(i) $(+2) + (+3)$ (ii) $(+3) + (+3)$ (iii) $(+4) + (+1)$ (iv) $(+5) + (+3)$

• The sum of two negative integers

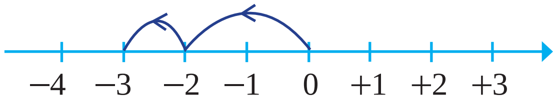
Let us consider adding directed numbers which are negative integers by using the number line.

Let us find the value of $(-2) + (-1)$ using the number line.

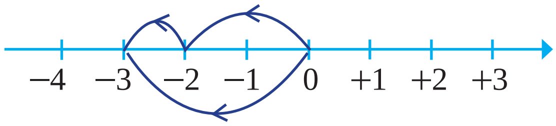
First, starting from 0, let us go two units towards the left along the number line.



Next, from this point, let us go one unit towards the left along the number line.



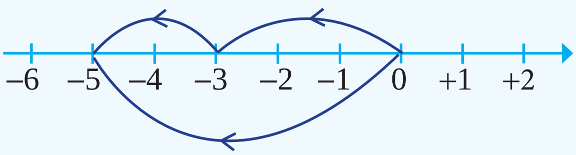
The directed number denoted by the position at which we finally stop is the answer.



$(-2) + (-1) = (-3)$

Example 1

Find the value of $(-3) + (-2)$ using the number line.



The final position is five units to the left of 0.

$\therefore\ (-3) + (-2) = (-5)$

Exercise 8.2

Find the value using the number line.

(i) $(-4) + (-1)$ (ii) $(-2) + (-2)$ (iii) $(-2) + (-3)$

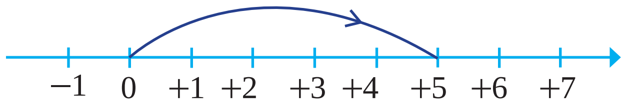
(iv) $(-1) + (-3)$ (v) $(-3) + (-3)$ (vi) $(-4) + (-2)$

• The sum of a positive integer and a negative integer

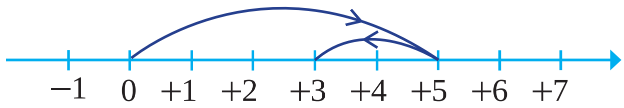
Now let us consider adding a positive integer and a negative integer.

Let us find the value of $(+5) + (-2)$ using the number line.

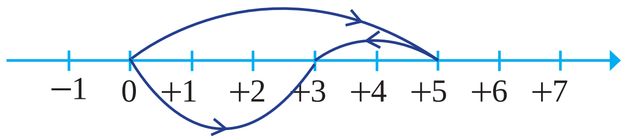
First, starting from 0, let us go five units towards the right along the number line.



Next, from this point, let us go two units towards the left along the number line.



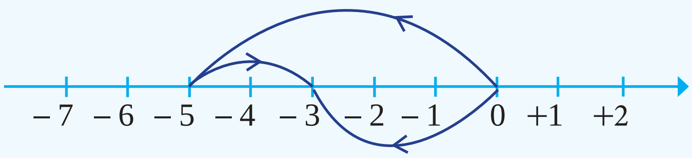
The directed number denoted by the position at which we finally stop is the answer.



$(+5) + (-2) = (+3)$

Example 1

Find the value of $(-5) + (+2)$ using the number line.



$(-5) + (+2) = (-3)$

Since the final position is three units to the left of 0, the number (-3) relevant to this position is the answer.

Exercise 8.3

Find the value using the number line.

(i) $(+3) + (-1)$	(ii) $(-4) + (+6)$	(iii) $(-7) + (+2)$
(iv) $(+2) + (-5)$	(v) $(+1) + (-1)$	(vi) $(-3) + (+3)$

8.3 Adding integers without using the number line

• Finding the sum of two integers

Let us consider the examples related to adding two positive integers that were studied in the previous section.

Using the number line we obtained previously that,

$(+2) + (+1) = (+3)$ and
$(+3) + (+2) = (+5)$.

$$(+2) + (+1) = (+3) \qquad\qquad (+3) + (+2) = (+5)$$
$$2 + 1 = 3 \qquad\qquad 3 + 2 = 5$$

- When adding two positive integers, add the two numbers without considering the signs.
- Place the positive sign in the final answer.

Let us now reconsider the examples related to adding two negative integers that were studied in the previous section.

Using the number line we obtained previously that

$(-2) + (-1) = (-3)$ and
$(-3) + (-2) = (-5)$.

Let us consider $(-2) + (-1) = (-3)$

- Without considering the signs of the two directed numbers, obtain their sum.

  $2 + 1 = 3$
- Then write the answer with the negative sign. Therefore the answer is -3.

When adding two negative directed numbers, add the two numbers without considering the negative sign and then write the answer with the negative sign.

Example 1

Simplify:

(i) $(+4) + (+6)$ (ii) $(+11) + (+3)$ (iii) $(-5) + (-2)$ (iv) $(-4) + (-1)$

(i) $(+4) + (+6) = (+10)$ (ii) $(+11) + (+3) = (+14)$

(iii) $(-5) + (-2) = (-7)$ (iv) $(-4) + (-1) = (-5)$

Exercise 8.4

Simplify.

(i) $(+3) + (+8)$ (ii) $(-7) + (-3)$ (iii) $(+12) + (+4)$

(iv) $(-9) + (-16)$ (v) $(-20) + (-13)$ (vi) $(+17) + (+13)$

(vii) $(-11) + (-29)$ (viii) $(+2) + (+8)$ (ix) $(-3) + (-10)$

• Finding the sum of a positive integer and a negative integer

Using the number line we obtained previously that,

$(+5) + (-2) = (+3)$ and

$(-5) + (+2) = (-3)$.

We can find the sum of a positive integer and a negative integer as follows.

Let us consider $(-8) + (+5)$.

- Without considering the signs of the two directed numbers, obtain their difference. $8 - 5 = 3$
- From the two directed numbers (-8) and $(+5)$, the number which is further away from 0 on the number line is (-8). Its sign is negative.
- Therefore the answer is -3.

$(-8) + (+5) = (-3)$

When adding two directed numbers of opposite signs (positive and negative), obtain their difference without considering the signs, and write the answer with the sign of the directed number which is further away from 0 on the number line.

Example 1

Simplify $(+8) + (-3)$

$8 - 3 = 5$

From the two directed numbers $(+8)$ and (-3), the number which is further away from 0 on the number line is $(+8)$. Its sign is positive.

$(+8) + (-3) = (+5)$

Example 2

Simplify $(+4) + (-10)$

$10 - 4 = 6$

From the two directed numbers $(+4)$ and (-10), the number which is further away from 0 on the number line is (-10). Its sign is negative.

$(+4) + (-10) = (-6)$

Exercise 8.5

(1) Evaluate the following.

(i) $(+7) + (-2)$	(ii) $(-10) + (+4)$	(iii) $(-3) + (+6)$
(iv) $(-5) + (+9)$	(v) $(-11) + (+4)$	(vi) $(-4) + 0$
(vii) $(+9) + (-8)$	(viii) $(+7) + (-15)$	(ix) $(+5) + (-6)$
(x) $(-7) + (+5)$	(xi) $(+8) + (-10)$	(xii) $(-9) + (+4)$

8.4 Adding directed numbers

We have so far considered the addition of directed numbers which are integers. Now let us consider the addition of any two directed numbers.

The methods that were used above to add integers are used here too.

Example 1

Add the following directed numbers.

(i) $(+\frac{1}{2}) + (+\frac{1}{2})$

Without considering the signs of the two directed numbers, obtain their sum.

$$\frac{1}{2} + \frac{1}{2} = 1$$

Place the positive sign in the final answer.

$$(+\frac{1}{2}) + (+\frac{1}{2}) = +1$$

(ii) $(-\frac{2}{7}) + (-\frac{4}{7})$

Without considering the signs of the two directed numbers, obtain their sum.

$$\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$$

Place the negative sign in the final answer.

$$(-\frac{2}{7}) + (-\frac{4}{7}) = (-\frac{6}{7})$$

(iii) $(+ 7.2) + (+ 1.3) = (+ 8.5)$

(iv) $(- 6.9) + (+ 2.5) = (- 4.4)$

Exercise 8.6

Evaluate the following.

(i) $(+\frac{3}{5}) + (+\frac{1}{5})$ (ii) $(-\frac{4}{7}) + (-\frac{1}{7})$ (iii) $(+\frac{2}{3}) + (+\frac{1}{3})$

(iv) $(-2) + (-\frac{1}{2})$ (v) $(-8.1) + (-1.3)$ (vi) $(-3.6) + (-1.8)$

(vii) $(+4) + (-2.5)$ (viii) $(-5) + (-3.7)$ (ix) $(-\frac{4}{8}) + (-\frac{3}{8})$

(x) $(-2.6) + (+6.5) + (-4.3)$ (xi) $(+5.7) + (-3.9) + (+1.4)$

Miscellaneous Exercise

(1) Fill in the blanks.

(i) $(+ 8) + (- 1) =$ (.....) (ii) $(+ 11) + (- 12) =$ (.....)

(iii) $(- 4) + (- 11) =$ (.....) (iv) $(-\frac{7}{9}) + (-\frac{5}{9}) =$ (.....)

(v) $(-\frac{8}{11}) + (-\frac{3}{11}) =$ (.....) (vi) $(+ 8.95) + (+ 2.97) =$ (.....)

(vii) $(- 5.81) + (- 2.25) =$ (.....) (viii) $(- 6.57) + (+ 11.21) =$ (.....)

(ix) $(-\frac{4}{13}) + (-\frac{7}{13}) =$ (.....) (x) $(+ 3.52) + (- 2.51) =$ (.....)

(2) The ground floor of a building has been named Floor 0 and the floors above it have been named 1, 2, 3, … respectively, while the floors below it have been named -1, -2, -3, ... respectively.

(i) If a person in Floor 7 climbs up a further 5 floors, which floor will he be in?

(ii) If a person in Floor -1 descends a further 2 floors, which floor will he be in?

(iii) If a person in Floor 8 descends 3 floors, which floor will he be in?

(iv) If a person in Floor 2 descends 4 floors, which floor will he be in?

(3) The temperature at 6.00 a.m. in Moscow on a certain day was recorded as -4.7° C, while the temperature at 4.00 p.m. on the same day was increased by 12^{0} C. Find the temperature in Moscow at 4.00 p.m.

Summary

- All numbers that are written with a positive or negative sign to indicate not only their magnitude but also one of two directions which are opposite to each other are called directed numbers.
- When adding two directed numbers of the same sign, add the numbers without considering the sign, and then include the sign with the answer.
- When adding two directed numbers of opposite signs (positive and negative), obtain their difference without considering the signs, and write the answer with the sign of the directed number which is further away from 0 on the number line.

9

Angles

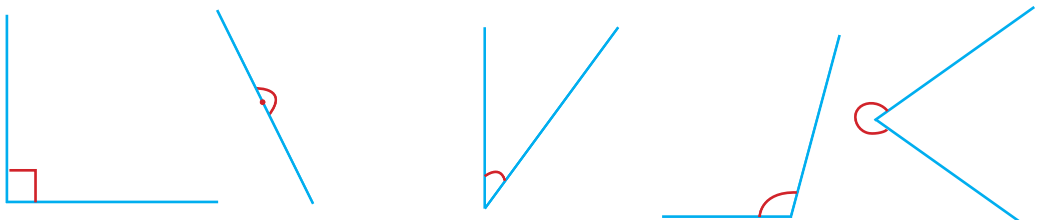
By studying this lesson you will be able to

- identify the dynamic or static nature of an angle,
- name angles,
- measure and draw angles using the protractor, and
- classify angles based on their magnitude.

9.1 Angles

You learnt in grade 6 that an angle is created when two straight line segments meet each other.

Below are a few types of angles we identified.



Right angle | Straight angle | Acute angle | Obtuse angle | Reflex angle

Do the following review exercise to recall the facts you have learnt about angles.

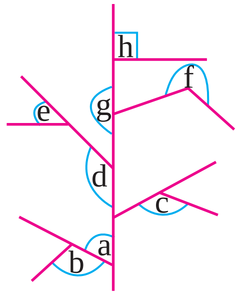
Review Exercise

(1) Choose the figures that are angles and write down the corresponding letters.

(a) (b) (c) (d)

(2) Identify the angles in the figure below and complete the table.

Angle	Type of Angle	Angle	Type of Angle
a		e	
b		f	
c		g	
d		h	

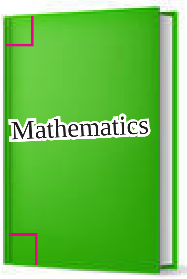


(3) Draw an angle of each type on a square ruled paper. Write the type of angle next to the corresponding figure.
Acute angle, Right angle, Obtuse angle, Straight angle, Reflex angle

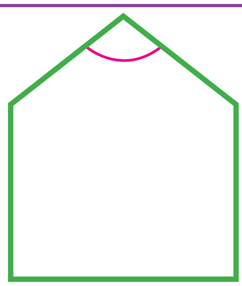
9.2 The dynamic or static nature of an angle

Let us investigate more on angles.

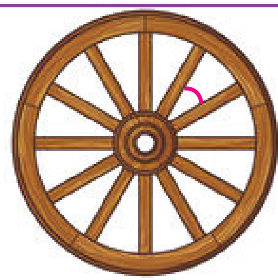
If we observe our surroundings, we can identify many angles. A few examples are given below.



Angle between the edges of a cover of a book



Angle between two parts of the roof.

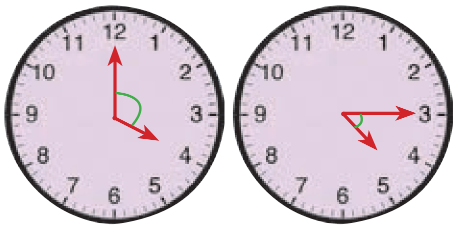


Angles between the adjacent spokes of a wheel

A common property of the above angles is that their magnitude does not change.

- If the magnitude of an angle does not change, then it is static in nature.
- So the angles in the above figures are static in nature.
- Note that the magnitude of the angle between two spokes of a wheel does not change, even when the wheel is turning.

Let us now consider some situations that involve rotation.

		
The angle between the hour hand and the minute hand of a clock changes in magnitude with time. The figure shows this angle at 4 p.m. and 4.15 p.m.	The angle between the two blades of a pair of scissors changes when it is used for cutting.	The angle between the top edge of a door and a door frame changes when the door is being opened or closed.

In the examples given above, let us consider the **arms** of the angle involved.

We see that there is a rotation of both arms or of one of them. Therefore, the magnitude of the angle changes. This is the **dynamic nature** of such an angle.

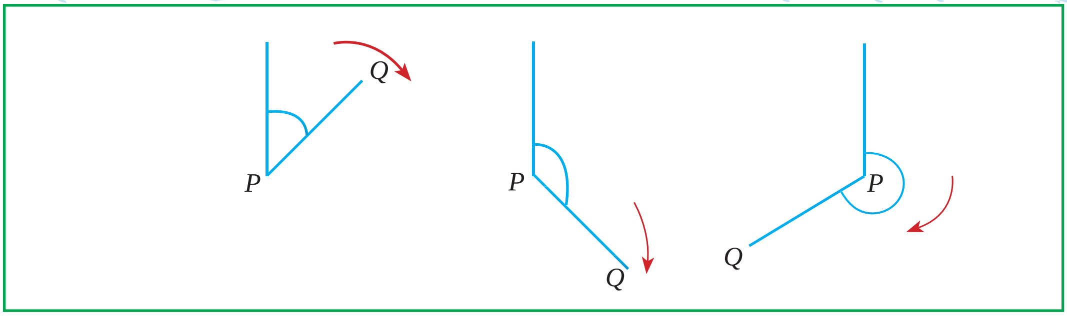
Let us understand the dynamic nature of an angle further by doing the following activity.

Activity 1

Step 1 - Take a fresh green ekel and bend it into two parts at the centre, taking care not to break it.

Step 2 - Overlap the two parts of the ekel and place it on a table. Hold one part tightly on the table.

Step 3 - In your exercise book, draw several situations that are obtained by rotating the other part on the table.
A few such situations that can be obtained are shown below.



- You can see that the magnitude of the angle between the two parts of the ekel changes. That is, this angle is dynamic in nature.
- When both parts of the ekel are rotated too the magnitude of the angle between the two parts changes.

A rotation which is in the same direction as the rotation of the arms of a clock is defined as a clockwise rotation. A rotation which is in the opposite direction to that of the rotation of the arms of a clock is defined as an anticlockwise rotation.

Exercise 9.1

(1) (i) Write down 3 instances where you can observe angles which are dynamic in nature in your surrounding environment.

(ii) Write down 3 instances where you can observe angles which are static in nature in your surrounding environment.

(2) (i) Give an example of an angle which is static in nature where the positions of the arms of the angle are fixed.

(ii) Give an example of an angle which is static in nature where there is a change in the positions of the arms of the angle.

(iii) Give an example of an angle which is dynamic in nature where there is a change in the position of only one arm of the angle.

(iv) Give an example of an angle which is dynamic in nature where there is a change in the positions of both arms of the angle.

9.3 Naming Angles

Let us now consider how angles are named.

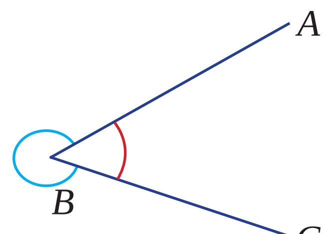


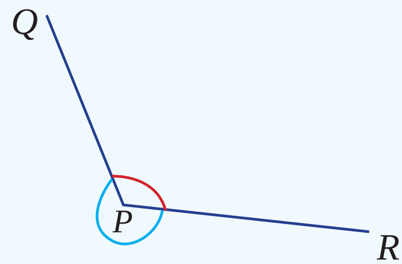
Figure I

- In Figure I, two angles have been created by the straight line segments *AB* and *BC* meeting.
- The straight line segments *AB* and *BC* are defined as the "arms of the angle". The point *B* where *AB* and *BC* meet is defined as the "vertex of the angle".
- The magnitude of the angle which is indicated in red is less than that of a straight angle; that is, less than the magnitude of two right angles.
- The magnitude of the angle indicated in blue is greater than that of a straight angle.
- The angle indicated in red is named as angle *ABC* and is written as , $A\hat{B}C$ or $C\hat{B}A$
- Here we write the letter which indicates the vertex in the middle and the other two letters beside it.
- The angle indicated in blue is named as the reflex angle *ABC* and is written as reflex angle $A\hat{B}C$ or reflex angle $C\hat{B}A$.
- In some books angle *ABC* is written as $\sphericalangle ABC$.

Example 1

Draw the angles with the straight line segments *PQ* and *PR* as their arms. Name the two angles.

↳ Since *P* is common to both arms, *P* is the vertex of the angles. Therefore the angle indicated in red is $Q\hat{P}R$ and the angle indicated in blue is the reflex angle $Q\hat{P}R$.



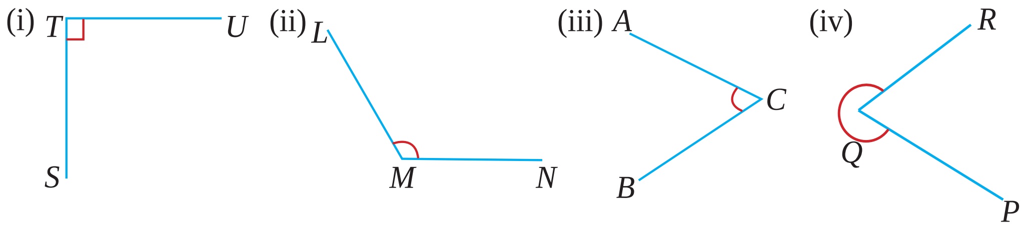
Example 2

Write down the vertex and the arms of $D\hat{E}F$.

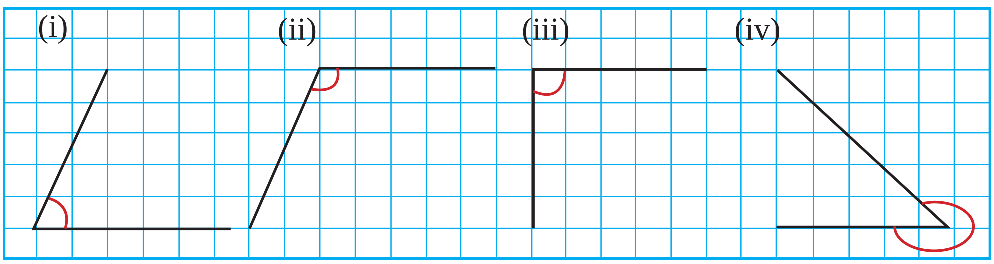
Since the letter in the middle of $D\hat{E}F$ is E, the vertex of the angle is E and the arms of the angle are ED and EF.

Exercise 9.2

(1) Write down the arms and the vertex of each of the angles given below.



(2) Copy each of the angles given below and name them using letters of the English alphabet.



(3) Draw and name an angle of your choice on a square ruled paper.

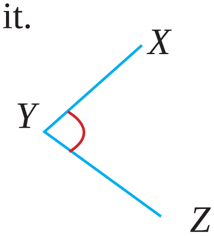
(4) Draw an obtuse angle with arms XY and YZ on a square ruled paper.

(5) Draw an angle and name it $D\hat{E}F$. Name its arms and its vertex.

(6) Draw a reflex angle and name it.

(7) Draw a right angle on a square ruled paper and name it.

(8) Prabath has written the angle in the figure as $X\hat{Y}Z$. Sumudu has written it as $Z\hat{Y}X$. Kasun says that both Prabath and Sumudu are correct. Do you agree with Kasun? Explain your answer.



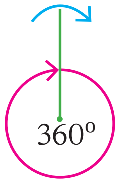
9.4 Measuring angles

There are standard units and instruments to measure distance, mass, time and the volume of a liquid. You learnt about these in grade 6.

Now let us learn about a standard unit and an instrument used to measure angles.

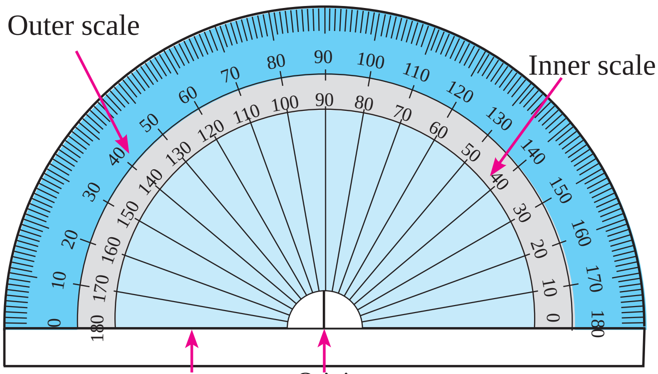
The standard unit used to measure angles is **degrees**. One degree is written as 1°.

The angle that is formed when a straight line segment completes one full circle by rotating about a point is 360°.



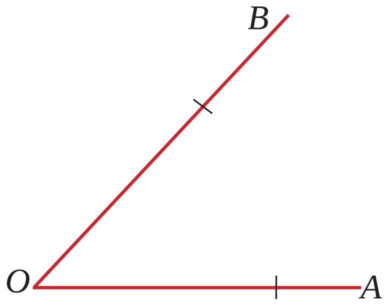
The instrument used to measure angles is made of one half of a full circle. It is called a "**protractor**". The figure of a protractor is shown below. It is numbered from 0° to 180° clockwise and anticlockwise. The line indicated by 0 - 0 is called the "base line".

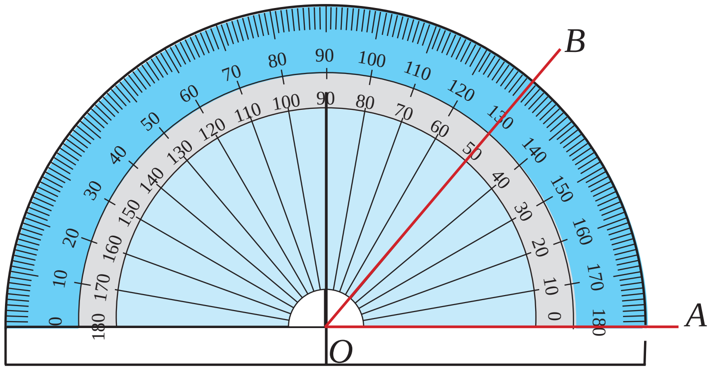
There are two scales indicated in the protractor. They are the inner scale and the outer scale.



The long line segments on the outer scale are marked as 0, 10, 20, …, 180. The gap between every pair of long line segments is again divided into 10 similar parts using short line segments. As indicated in the figure, the magnitude of the angle between two long line segments is 10°.

Let us now see how we can use the protractor to measure the angle $A\hat{O}B$ in the figure.





Place the protractor on the figure such that the origin and the base line coincide with the vertex O and the arm OA respectively.

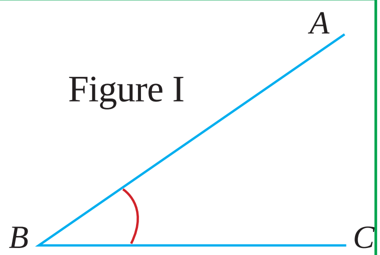
Then the arm OB coincides with the line indicated by 50^{o} in the inner scale (Note that OA coincides with 0^{o} on this scale). Therefore the magnitude of the angle $A\hat{O}B$ is 50^{o}, and we write $A\hat{O}B = 50^{o}$.

By observing this figure, we see that an angle of 1^{o} is a small angle which is difficult to draw.

Activity 2

Step 1 - In your exercise book, draw an angle similar to the one in Figure I using a ruler.

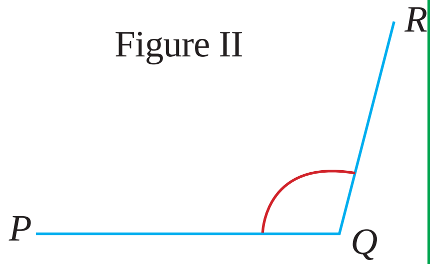
Figure I



Step 2 - Measure the magnitude of the angle drawn, and write it inside the space margined by AB, BC and the red arc.

Step 3 - Draw an angle similar to the one in Figure II below, measure the magnitude of the angle and write it down as done in step 2.

Figure II

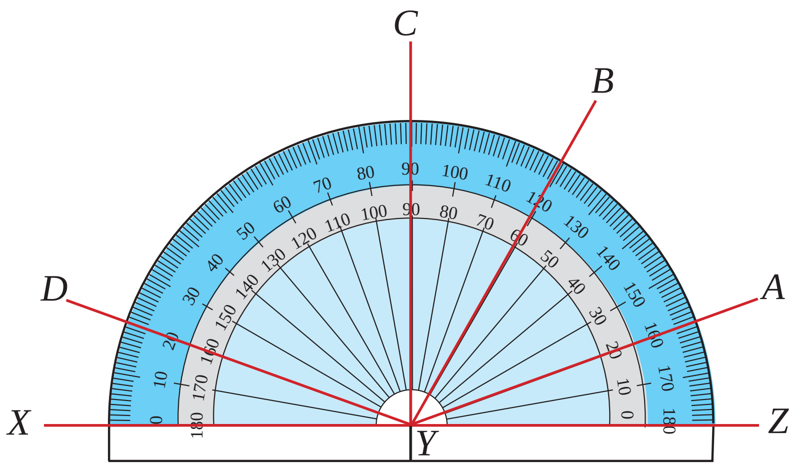


Exercise 9.3

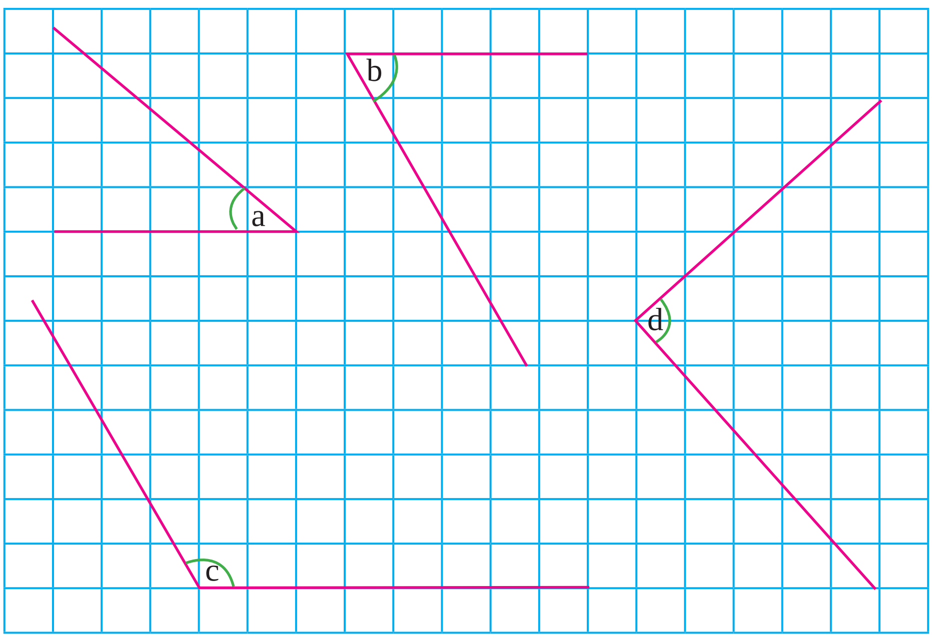
(1) Write down the magnitude of each angle using the given figure.

(i) $X\hat{Y}Z$ (ii) $Z\hat{Y}A$ (iii) $X\hat{Y}C$ (iv) $B\hat{Y}Z$

(v) $X\hat{Y}B$ (vi) $C\hat{Y}Z$ (vii) $X\hat{Y}A$ (viii) $Z\hat{Y}D$

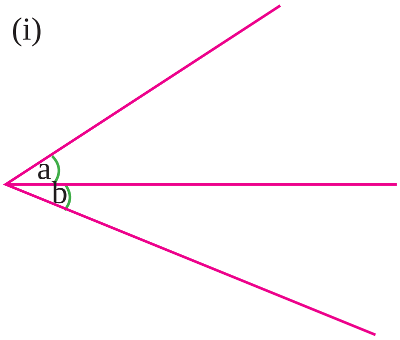


(2) Draw each of the angles below on a square ruled paper. Measure and write the magnitude of each angle.

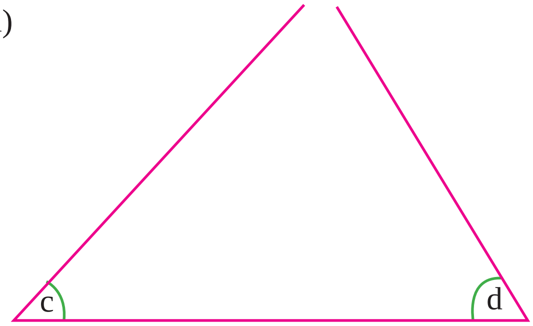


(3) Draw the following figures in your exercise book. Measure and write down the magnitude of each of the angles indicated by the English letters.

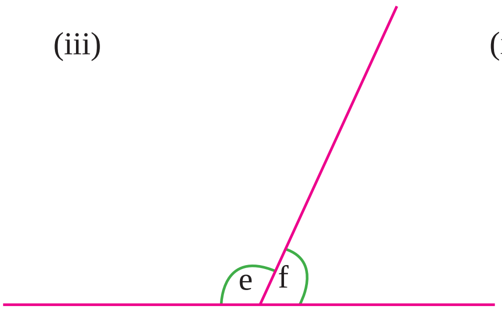
(i)



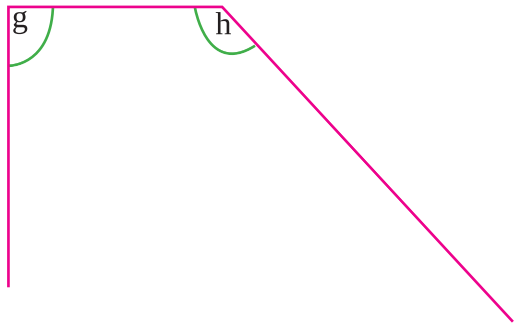
(ii)



(iii)



(iv)



9.5 Drawing angles with given magnitude

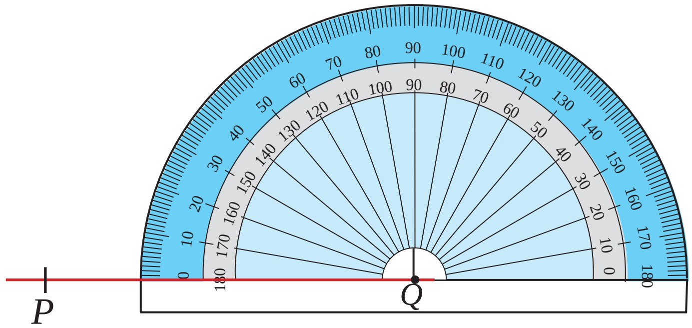
Let us now draw angles when their magnitude is given.

Activity 3

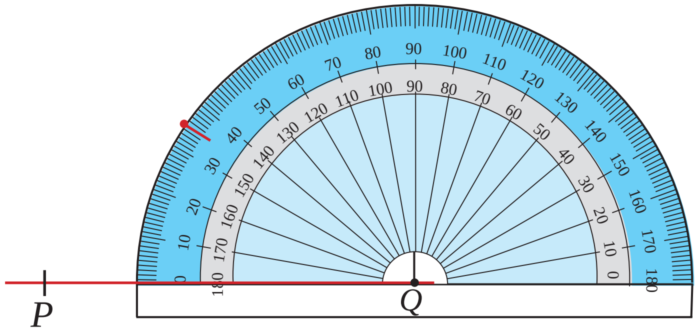
Performing the steps given below, draw the angle $P\hat{Q}R = 35^{o}$

Step 1 - Using the ruler, draw a straight line segment and name it PQ.

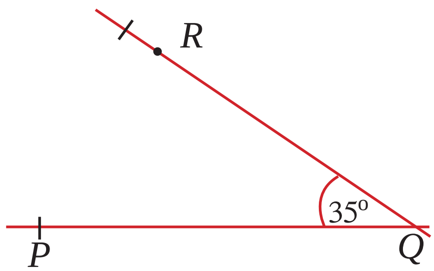
Step 2 - Since the vertex of the angle is Q, place the protractor so that its origin and the base line coincide with Q and PQ respectively.



Step 3 - Now find 35° in the outer scale. Place a dot mark on the paper at 35^0.

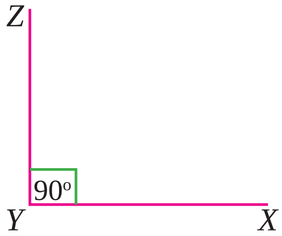


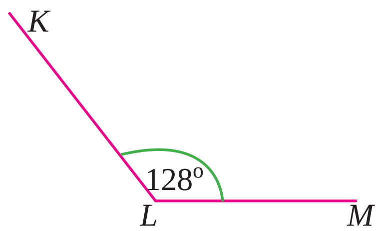
Step 4 - Remove the protractor. Name the dot marked in step 3 as *R*. Now draw a straight line from *Q* to *R*. Write the magnitude of the angle $P\hat{Q}R$ as 35^0.



As above, draw the following:

(i) $X\hat{Y}Z = 90^{o}$ (ii) $K\hat{L}M = 128^{o}$





Exercise 9.4

(1) Draw the following angles.

(i) $A\hat{B}C = 48^\circ$ (ii) $P\hat{Q}R = 90^\circ$ (iii) $K\hat{L}M = 130^\circ$ (iv) $X\hat{Y}Z = 28^\circ$

(2) (i) Draw a straight line segment and name it PQ.

(ii) Draw the arm PR such that $Q\hat{P}R = 82^\circ$.

(iii) Draw the arm QS such that $P\hat{Q}S = 43^\circ$.

(3) (i) Draw any triangle you like and name it ABC.

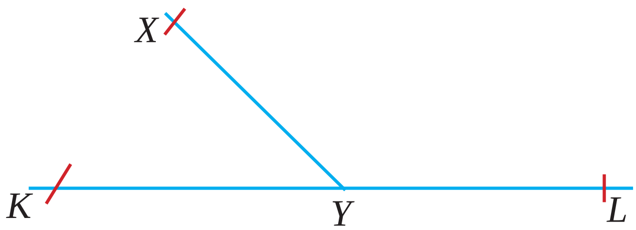
(ii) Measure $A\hat{B}C$, $B\hat{C}A$ and $C\hat{A}B$ and write their magnitudes separately.

(iii) Obtain the value of $A\hat{B}C + B\hat{C}A + C\hat{A}B$ using the measured values.

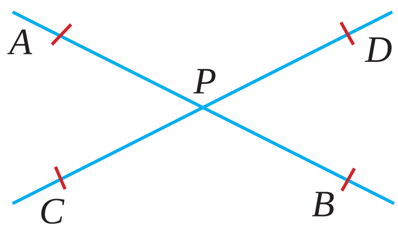
(4) (i) Draw two straight line segments KL and XY so that they meet each other at Y.

(ii) Measure and write down the magnitudes of $K\hat{Y}X$ and $X\hat{Y}L$

(iii) Obtain $K\hat{Y}X + X\hat{Y}L$.



(5) (i) As given in the figure, draw two straight line segments AB and CD so that they intersect each other.



(ii) Measure and write the magnitudes of $A\hat{P}C$, $C\hat{P}B$, $B\hat{P}D$ and $D\hat{P}A$ separately.

(iii) Write the relationship between $A\hat{P}C$ and $B\hat{P}D$.

(iv) Write the relationship between $A\hat{P}D$ and $C\hat{P}B$.

(6) Dasun says that the angle in Figure (II) is larger than the angle in Figure (I). Do you agree? Explain your answer.

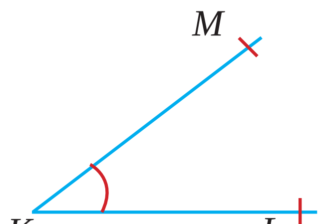


Figure I

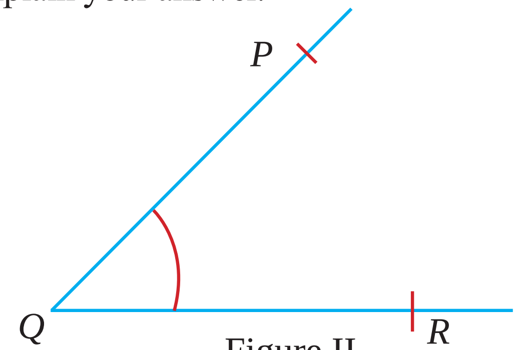


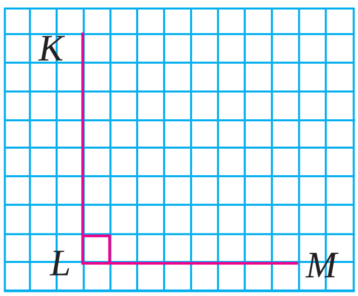
Figure II

9.6 Classification of angles

We learnt in grade 6 to classify angles using a right angle. Magnitude of a right angle is 90°. We can classify angles by comparing them with 90°.

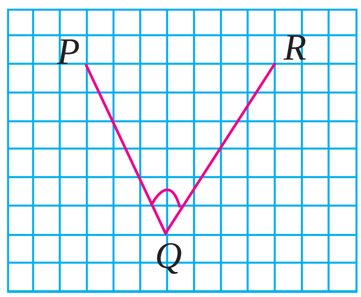
Right Angles

Any angle of magnitude 90° is called a "right angle". $K\hat{L}M$ is a right angle.



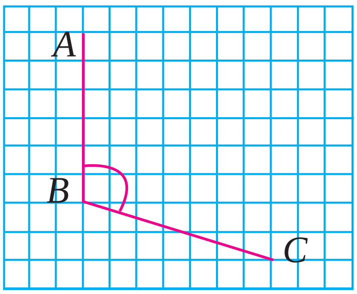
Acute Angles

Any angle of magnitude less than 90° is called an "acute angle". $P\hat{Q}R$ is an acute angle.



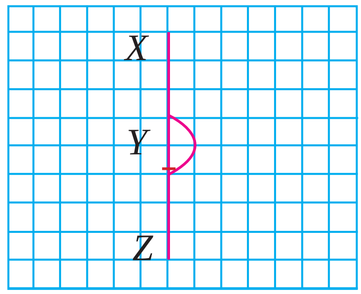
Obtuse Angles

Any angle of magnitude greater than 90° but less than 180° (that is an angle between 90°and 180°) is called an "obtuse angle". $A\hat{B}C$ is an obtuse angle.



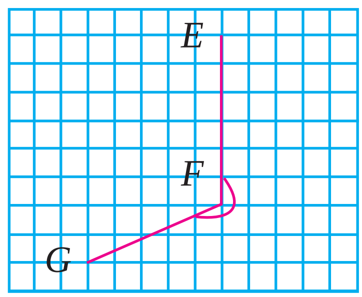
Straight Angles

Any angle of magnitude 180° is called a "straight angle". $X\hat{Y}Z$ is a straight angle.



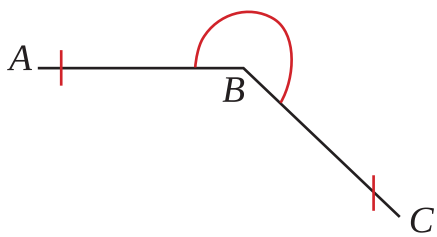
Reflex Angles

Any angle of magnitude between 180°and 360° is called a " reflex angle". $E\hat{F}G$ is a reflex angle.



9.7 Measuring and Drawing Reflex angles

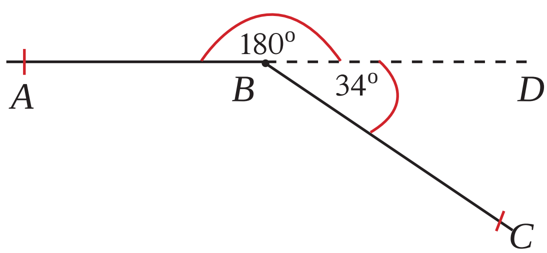
The figure shows the reflex angle $A\hat{B}C$.This angle cannot be measured directly using a protractor. So let us see how we can measure this reflex angle.



Method I :-

Let us use the ruler to extend AB and obtain the straight angle $A\hat{B}D$.

That is, $A\hat{B}D = 180^o$.

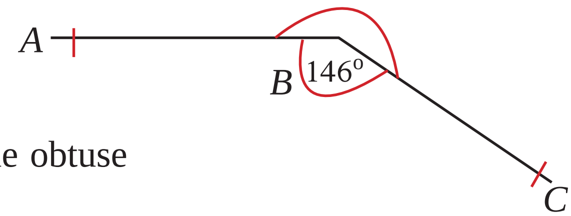


Now let us measure $D\hat{B}C$ using the protractor. We will then obtain $D\hat{B}C = 34^{o}$.

Since the reflex angle $A\hat{B}C = A\hat{B}D + D\hat{B}C$
the reflex angle $A\hat{B}C = 180^{o} + 34^{o} = 214^{o}$.

Method II :-

Measure the obtuse angle $A\hat{B}C$.
It is equal to146°.
Since the reflex angle $A\hat{B}C$ + the obtuse angle $A\hat{B}C = 360^{o}$



The reflex angle $A\hat{B}C = 360^{o} - 146^{o}$
$= 214^{o}$

Let us now see how to draw reflex angles.

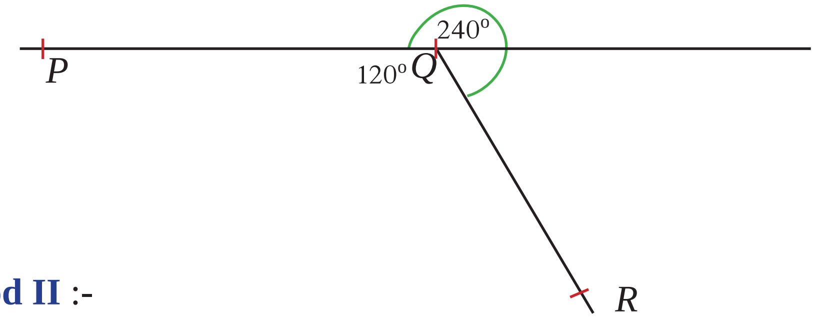
Activity 4

Draw the reflex angle $P\hat{Q}R = 240^{o}$ according to the following steps.

Step 1 - Draw the straight line segment PQ.

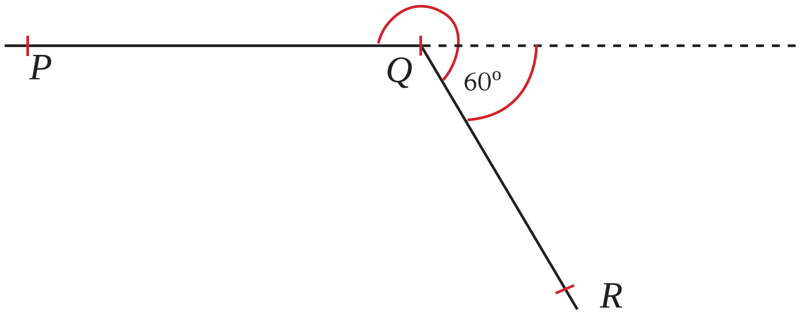
Step 2 - Calculate the magnitude of the obtuse angle $P\hat{Q}R$.
$P\hat{Q}R = 360^{o} - 240^{o} = 120^{o}$

Step 3 - Draw the angle $P\hat{Q}R = 120^{o}$. Now mark the reflex angle 240°.



Method II :-

Step 4 - By drawing an angle of 60° (That is, $240^{\circ} - 180^{\circ}$) on the straight angle appropriately, we can obtain the reflex angle 240°.



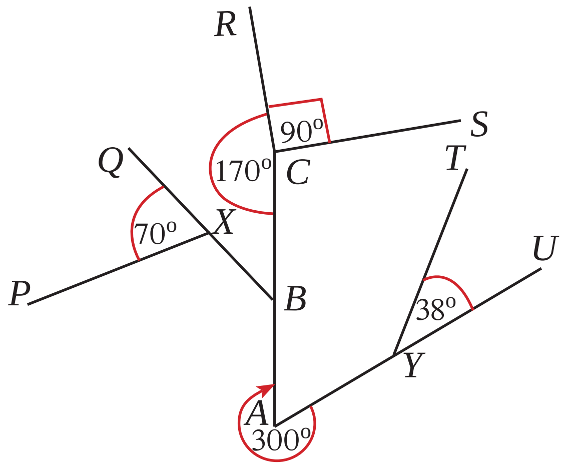
Exercise 9.5

(1) Copy the two groups (*a*) and (*b*) in your exercise book. Join each angle and its type with a straight line.

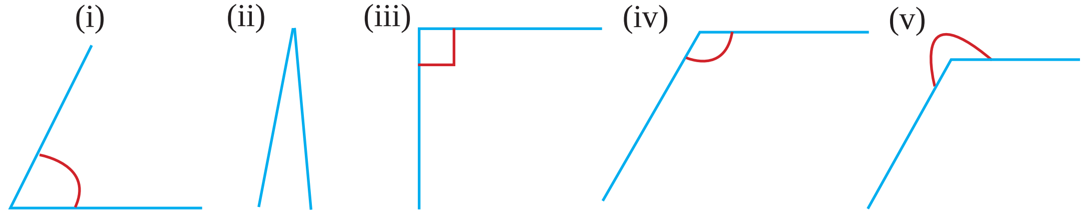
Group (*a*) (Magnitude of the angle)	Group (*b*) (Type of angle)
18°	Straight angle
135°	Right angle
180°	Acute angle
255°	Obtuse angle
90°	Reflex angle

(2) Using the information in the figure, write down the type of each of the angles given below.

(i) $P\hat{X}Q$ (ii) $B\hat{C}R$ (iii) $S\hat{C}R$ (iv) $T\hat{Y}U$ (v) $B\hat{A}Y$



(3) Choose and write down the most appropriate magnitude for each of the angles below, from the values given in brackets.



(25°, 65°, 10°) (1°, 80°, 15°) (50°, 90°, 180°) (360°, 120°, 180°) (185°, 240°, 350°)

(4) Draw the following reflex angles using the protractor.

(i) $A\hat{B}C = 300°$ (ii) $P\hat{Q}R = 195°$ (iii) $M\hat{N}O = 200°$

(iv) $K\hat{L}M = 243°$ (v) $X\hat{Y}Z = 310°$

Summary

- The standard unit used to measure angles is degrees. One degree is written as 1°.
- Any angle of magnitude less than 90° is called an "acute angle".
- Any angle of magnitude 90° is called a "right angle".
- Any angle of magnitude greater than 90° but less than 180° (that is an angle between 90° and 180°) is called an "obtuse angle".
- Any angle of magnitude 180° is called a "straight angle".
- Any angle of magnitude between 180° and 360° is called a "reflex angle".

Revision Exercise - I

(1) (a) Simplify the following.

(i) $15 + 13 + 12$ (ii) $18 - 12 + 6$ (iii) $9 + 6 - 8$

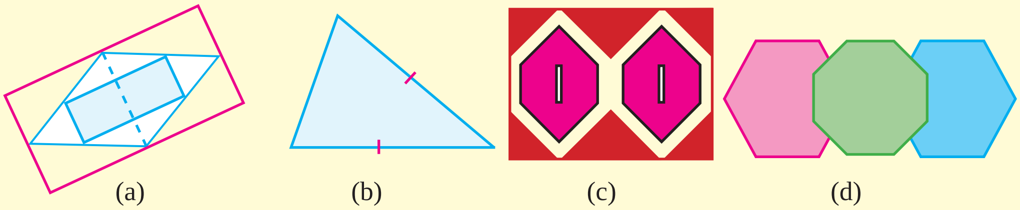
(iv) $8 \times 7 - 12$ (v) $7 \times 3 + 5$ (vi) $24 - 18 \div 3$

(vii) $15 + 18 \div 3$ (viii) $16 + 5 \times 3$ (ix) $15 - 9 \div 3$

(b) Hasintha says "when we simplify $91 - 35 \div 7$, we get 8 as the answer". Explain why Hasintha's answer is incorrect.

(2) (i) What is a bilaterally symmetric plane figure?

(ii) Write the number of axes of symmetry in each of the symmetric figures given below.

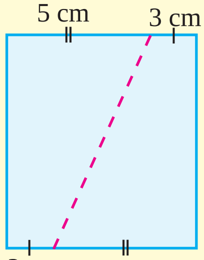


(a) (b) (c) (d)

(iii) Draw the following symmetric figures in your square ruled exercise book. Draw their axes of symmetry and name them.

(a) A rectilinear plane figure with only one axis of symmetry
(b) A rectilinear plane figure with only two axes of symmetry
(c) A rectilinear plane figure with more than two axes of symmetry

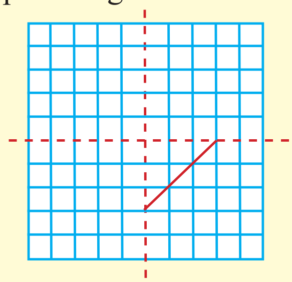
(iv)



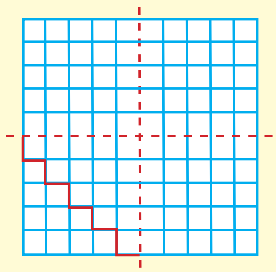
If the plane figure is cut along the dotted line, then it will be divided into two parts which coincide with each other.

Is the figure bilaterally symmetric about this line? Explain your answer giving reasons.

(v) Copy each of the following figures onto a square ruled paper and complete each figure such that the two dotted lines become axes of symmetry of the completed figure.



(a)



(b)

(3) (i) Set A is given below by listing its elements.

$A = \{2, 3, 5, 7\}$

Write A using a common property of its elements.

(ii) Re-write P = {factors of 12} by listing its elements.

(iii) Let A = {multiples of 3 that lie between 8 and 20}

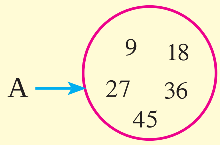
(a) Write A by listing its elements.

(b) Represent A in a Venn diagram.

(iv) Write the set represented by the Venn diagram,

(a) using a common property of its elements,

(b) by listing its elements.



(4) (i) Write the factors of 44.

(ii) Write the prime factors of 44.

(iii) Write 56 as a product of its prime factors.

(iv) Find the highest common factor of 18, 30, 42.

(v) Find the least common multiple of 18, 30, 42.

(5) (i) What is the digital root of 522?

(ii) Using the digital root, explain why 522 is divisible by 3.

(iii) Using the digital root explain why 522 is divisible by 9.

(iv) How do we find out without dividing a number whether it is divisible by 4 or not?

(v) $\boxed{4}$ $\boxed{3}$ $\boxed{2}$ $\boxed{1}$ are four numbers written on four cards. How many numbers which are divisible by 4 can be made using all these cards? Write down all such numbers.

(vi) If the number $53\square$ which has 3 digits is divisible by 9, then what is the digit in the units place?

(vii) If the number $53\square$ which has 3 digits is divisible by 6, then what is the digit in the units place?

(6) (a) (i) Find the value of 6^2.

(ii) Write all the factors of the number corresponding to the value found in (i).

(iii) There are only two prime factors among the factors written in (ii). Write down three more numbers where each of them has only two prime factors.

(iv) Write each of the three numbers as a power of a prime number.

(b) (i) Expand $a^2\ b^3$.

(ii) Evaluate $x^3\ y^2$ when $x = 5$ and $y = 4$.

(7) Write whether the following statements are true or false.

(i) Any multiple of 2 has only one prime factor.

(ii) Any number which can be written as a power of 2, has 2 as its only prime factor.

(iii) Any multiple of 3 has only one prime factor.

(iv) Any number which can be written as a power of 3, has only one prime factor.

(v) Any number which can be written as a power of 5, has 5 as its only prime factor.

(vi) The highest common factor of any two positive integers is less than or equal to their least common multiple.

(vii) The highest common factor of any two distinct prime numbers is 1.

(viii) The highest common factor of 12 and 13 is 1.

(8) (i) Explain, giving reasons whether AD 1892 is a leap year or not.

(ii) Explain, giving reasons whether AD 2100 is a leap year or not.

(iii) To which decade does the year AD 2100 belong to?

(9) (a) Add the following.

(i)

	years	months	days
	3	6	19
+	2	8	20

(ii)

	years	months	days
	16	9	21
+	7	3	9

(b) Subtract the following.

(i)

	years	months	days
	6	8	12
−	4	5	20

(ii)

	years	months	days
	5	7	19
−	2	9	25

(10) The fifth birthday of a child fell on 2002 - 08- 26. His mass was 20 kg and 700 g on that day.

(i) When was his birthday?

(ii) On his eighth birthday his mass was 30kg and 600g. What is the increase in his mass during the three years?

(iii) What was his age on 2012 - 03 - 25?

(iv) On 2012 - 03 - 25, the mass of the child was 12kg and 800g more than his mass on his fifth birthday. Find the mass of the child on 2012 - 03 - 25.

(11) (a) Using the number line, determine each of the following sums.

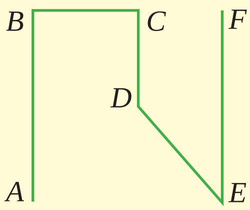
(i) $(-6)+(-4)$ (ii) $(-5)+(+5)$ (iii) $(+8)+(-9)$

(b) Simplify

(i) $(+4)+(-10)$ (ii) $(-9)+(+5)$ (iii) $(-8)+(-5)$

(iv) $(+\frac{1}{4})+(+\frac{1}{4})$ (v) $(-\frac{2}{7})+(-\frac{3}{7})$ (vi) $(-1.76)+(+0.36)$

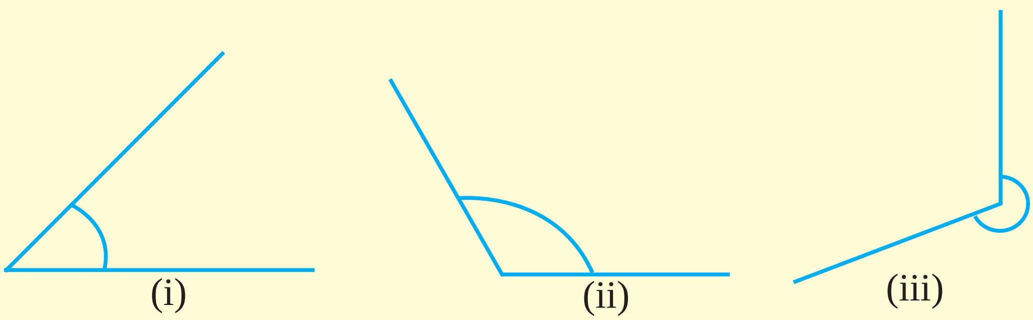
(12) (a)



Complete the table given below by considering a person whose journey starts at A and ends at F.

	Two roads travelled	Name the angle between the two roads	Name the vertex and the arms of the angle between the two roads	Name the type of angle between the two roads when classified according to its magnitude
(i)	A to C through B	....	....	....
(ii)	B to D through C	....	....	....
(iii)	C to E through D	....	....	....
(iv)	D to F through E	....	....	....

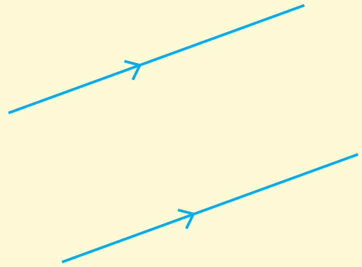
(b) Measure the magnitude of each angle given below using a protractor and write it down.



(c) Draw the following angles using the protractor and the ruler.

(i) $A\hat{B}C = 65^\circ$ (ii) $P\hat{Q}R = 130^\circ$ (iii) $M\hat{N}R = 145^\circ$

(13) (i) Two parallel lines are shown below. How far apart are they?

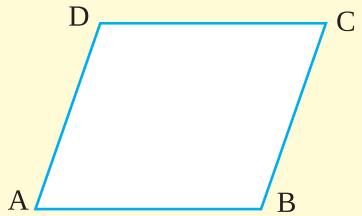


(ii) (a) Draw a straight line segment and name it *XY*.

(b) Mark a point *A* which is a distance of 4.8 cm from *XY*.

(c) Draw a straight line segment through the point *A* parallel to *XY*.

(iii) (a) Draw the parallelogram ABCD.



Draw parallel lines to diagonal AC throught B and D.

(14) (i) Nimal's birthday is 2002 -11 -25. Find Nimal's age in years, months and days on 2016 - 08 - 20.

(ii) Write the time that has elapsed between 12:35 of 2015 - 01 - 01 and 19:20 of 2015 - 02 - 05 in days, hours and minutes.

10

Fractions

(Part I)

By studying this lesson you will be able to

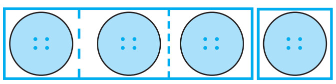
- identify mixed numbers and improper fractions, and
- convert a mixed number into an improper fraction and an improper fraction into a mixed number.

10.1 Fractions

Let us take the area bounded by the figure given below as one unit.

This unit has been divided into five equal parts, of which two have been coloured. As we have learnt previously, the amount coloured is $\frac{2}{5}$.

Similarly, if we take the four buttons shown below as one unit, we known that an amount of 3 buttons is $\frac{3}{4}$ of the amount of buttons there are.



Of the 25 children in a class, 13 are girls. When the number of girls in the class is written as a fraction of the total number of children in the class we obtain $\frac{13}{25}$. Here the total number of children in the class, that is 25, has been taken as one unit.

When a fraction is written numerically in this manner, the number below the line is called the **denominator** and the number above the line is called the **numerator**.

$$\frac{3 \leftarrow \text{Numerator}}{4 \leftarrow \text{Denominator}}$$

Numbers such as $\frac{2}{3}, \frac{3}{4}$ and $\frac{2}{5}$ which are smaller than one but larger than zero are called **proper fractions**. In a proper fraction, the numerator is always smaller than the denominator.

Proper fractions such as $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$, with the numerator equal to 1, are defined as **unit fractions**.

Any fraction can be written in terms of its corresponding unit fraction.

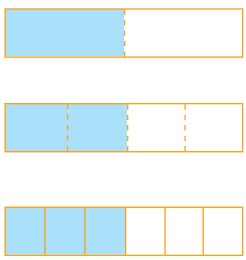
For example,

$\frac{2}{3}$ is two $\frac{1}{3}$ s.

$\frac{5}{17}$ is five $\frac{1}{17}$ s.

Next let us recall what we have learnt about equivalent fractions.

$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$



Let us consider these three figures. The amounts that have been coloured in these figures are equal. That is, the fractions $\frac{1}{2}, \frac{2}{4}$ and $\frac{3}{6}$ that are represented by these figures are equal to each other.

That is, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$.

We learnt in grade 6 that such fractions, which have denominators which are different to each other and numerators too which are different to each other, but which represent the same number are defined as equivalent fractions.

A fraction equivalent to a given fraction can be obtained by multiplying both the numerator and the denominator of the given fraction by a whole number (other than 0). Two such examples are given below.

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

A fraction equivalent to a given fraction can also be obtained by dividing both the numerator and the denominator of the given fraction by a whole number (other than 0) which divides the numerator and the denominator without remainder.

Let us find a fraction equivalent to $\frac{18}{24}$. Let us divide the numerator and the denominator of $\frac{18}{24}$ by 3 which divides 18 and 24 without remainder.

$$\frac{18}{24} = \frac{18 \div 3}{24 \div 3} = \frac{6}{8}$$

Do the following review exercise to recall the facts you have learnt about fractions.

Review Exercise

(1) Select the unit fractions from the following proper fractions and write them down.

$\frac{2}{3}, \frac{1}{7}, \frac{4}{15}, \frac{1}{3}, \frac{1}{100}$

(2) Fill in the blanks by selecting the suitable value from within the brackets.

(i) $\frac{3}{5}$ is ...... $\frac{1}{5}$ s (1, 2, 3)

(ii) $\frac{2}{7}$ is two ...... s. $(\frac{1}{2}, \frac{1}{7}, \frac{1}{5})$

(iii) Five $\frac{1}{6}$ s is equal to .... $(\frac{1}{30}, \frac{5}{6}, \frac{1}{5})$

(iv) $\frac{\boxed{...}}{12}$ is equivalent to $\frac{2}{3}$. (2 , 4, 8)

(3) Write down two equivalent fractions for each of the following fractions.

(i) $\frac{2}{3}$ (ii) $\frac{3}{5}$ (iii) $\frac{6}{8}$ (iv) $\frac{36}{48}$

(4) For each of the following fractions, write down the equivalent fraction with the smallest denominator.

$\frac{18}{30}, \frac{16}{24}, \frac{10}{35}$

(5) Write down the fractions $\frac{4}{7}, \frac{1}{7}, \frac{6}{7}, \frac{5}{7}$ in ascending order.

(6) Write down the fractions $\frac{7}{12}, \frac{5}{12}, \frac{2}{3}, \frac{1}{4}$ in descending order.

(7) If Sithmi obtained 21 marks out of a total of 25 marks for a test, express her marks as a fraction of the total marks.

(8) A vendor bought a stock of 50 mangoes of which 8 were spoilt.

(i) Express the number of spoilt fruits as a fraction of the total number of fruits.

(ii) Express the number of good fruits as a fraction of the total number of fruits.

10.2 Mixed numbers and improper fractions

1

$\frac{1}{2}$

A whole cake and exactly half of an identical cake are shown in the figure. When the whole cake is taken as a unit, it is expressed as 1 and the other part which is exactly half of such a cake, is expressed as $\frac{1}{2}$. Therefore, the total amount of cake in the figure is $1 + \frac{1}{2}$ times the whole cake. This is written as $1\frac{1}{2}$, and read as **one and a half**.

When a number which is the sum of a whole number and a proper fraction is written in this manner, it is defined as a **mixed number**. The whole number in the mixed number is called the **whole number part** and the proper fraction in it is called the **fractional part**.

$1\frac{1}{2}$, $1\frac{7}{8}$, $2\frac{2}{5}$ and $3\frac{1}{3}$ are examples of mixed numbers. The whole number part of the mixed number $2\frac{2}{5}$ is 2, and its fractional part is $\frac{2}{5}$.

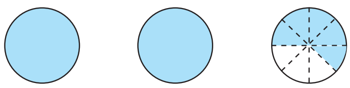
Let us write the mixed numbers represented in the following figures in the above manner.

1 $\quad \frac{2}{3}$

$$1+\frac{2}{3} = 1\frac{2}{3}$$

1 $\quad \frac{1}{4}$

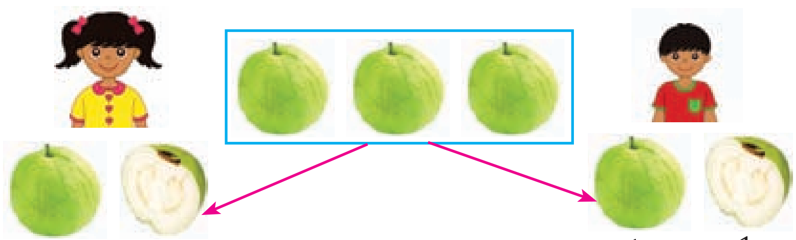
$$1+\frac{1}{4} = 1\frac{1}{4}$$



1 $\quad$ 1 $\quad \frac{5}{8}$

$$1+1+\frac{5}{8} = 2+\frac{5}{8} = 2\frac{5}{8}$$

Now let us consider one way of dividing three guavas of the same size equally between two children.

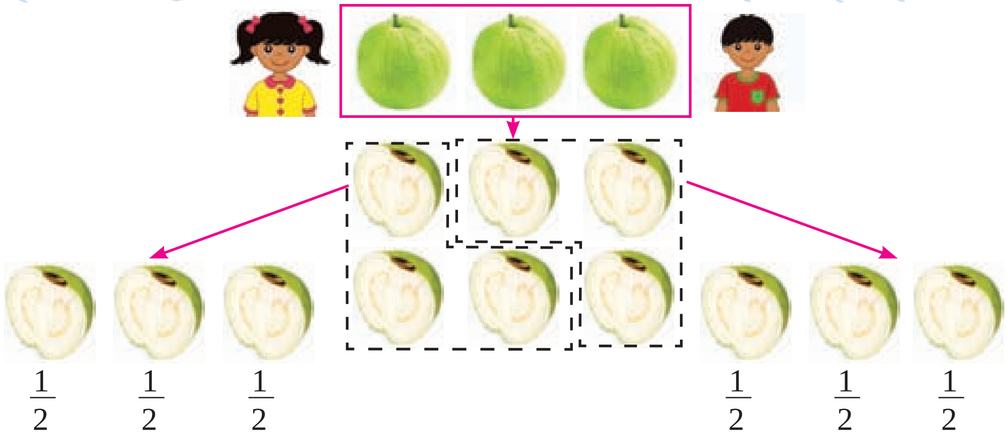


1 $\quad \frac{1}{2}$ $\qquad$ 1 $\quad \frac{1}{2}$

Here, each child receives one whole fruit and half of another fruit.

That is, the total amount that each child receives = $1+\frac{1}{2}$ fruits. This is written as $1\frac{1}{2}$.

Next, let us consider another way of dividing these three guavas equally between the two children.



3 halves $= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$.

(Divide each fruit into two equal parts)

3 halves $= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$.

Accordingly, each child gets 3 halves, which is an amount of $\frac{3}{2}$ guavas.

In both the above cases, each child receives the same amount of guava. Therefore, $\frac{3}{2} = 1\frac{1}{2}$.

The numerator of $\frac{3}{2}$ is greater than the denominator.

If the numerator of a fraction is greater or equal to the denominator, it is defined as an **improper fraction**.

We observed above that when three guavas are divided equally between two children, each child receives an amount of $\frac{3}{2}$ guavas. Therefore, $\frac{3}{2}$ represents the same value that is obtained when $3 \div 2$. That is, any proper fraction or improper fraction represents the number that is obtained when its numerator is divided by its denominator.

Example:: $\frac{2}{5} = 2 \div 5$ $\quad \frac{11}{3} = 11 \div 3$

$\frac{5}{2}$, $\frac{6}{3}$, $\frac{7}{5}$ and $\frac{11}{4}$ more examples of improper fractions.

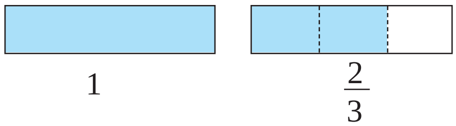
When the whole numbers 1, 2, 3 are expressed as $\frac{2}{2}$, $\frac{6}{3}$, and $\frac{15}{5}$ respectively, they too are considered as improper fractions.

Note that fractions with the numerator and denominator equal to each other are also considered as improper fractions.

10.3 Representing a mixed number as an improper fraction

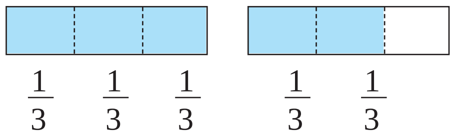
Let us find the shaded region in the figure using two methods.

First method



$1 + \frac{2}{3} = 1\frac{2}{3}$

Second method



The shaded region is five $\frac{1}{3}$s.

1 is equal to three $\frac{1}{3}$ s.

Five $\frac{1}{3}$ s. $= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$.

According to the above discussion, $1\frac{2}{3} = \frac{5}{3}$.

That is, the mixed number $1\frac{2}{3}$ can also be expressed as the improper fraction $\frac{5}{3}$.

Let us consider the mixed number $1\frac{3}{5}$.

Let us write the mixed number $1\frac{3}{5}$ as an improper fraction.

$$
\begin{aligned}
1\frac{3}{5} &= 1 + \frac{3}{5} \\
&= \frac{5}{5} + \frac{3}{5} \\
&= \frac{8}{5}
\end{aligned}
$$

Example 1

Express $2\frac{3}{4}$ as an improper fraction.

$$2\frac{3}{4} = 1 + 1 + \frac{3}{4}$$
$$= \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$$
$$= \frac{4+4+3}{4}$$
$$= \frac{11}{4}$$

Example 2

Express $3\frac{1}{2}$ as an improper fraction.

$$3\frac{1}{2} = 1 + 1 + 1 + \frac{1}{2}$$
$$= \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2}$$
$$= \frac{2+2+2+1}{2}$$
$$= \frac{7}{2}$$

Let us now consider an easy method of expressing a mixed number as an improper fraction. Let us consider the mixed number $1\frac{3}{5}$.

$$1 + \frac{3}{5} = \frac{5}{5} + \frac{3}{5}$$
$$= \frac{5+3}{5}$$
$$= \frac{(1 \times 5) + 3}{5} = \frac{8}{5}$$

- Multiply the whole number part in the mixed number, by the denominator of the fractional part and add this to the numerator of the fractional part.
- The value that is obtained is the value of the numerator of the improper fraction which is equal to the given mixed number.
- The denominator of the improper fraction is the same as the denominator of the fractional part of the mixed number.

Let us consider the following examples.

$$2\frac{3}{4} = \frac{(2 \times 4) + 3}{4} = \frac{8 + 3}{4} = \frac{11}{4}$$

$$3\frac{1}{2} = \frac{(3 \times 2) + 1}{2} = \frac{6 + 1}{2} = \frac{7}{2}$$

This process can be done mentally in one step; $7\frac{3}{8} = \frac{59}{8}$.

10.4 Expressing an improper fraction as a mixed number

Let us express $\frac{5}{3}$ as a mixed number .

Method I

$$\frac{5}{3} = \frac{3 + 2}{3}$$

$$= \frac{3}{3} + \frac{2}{3}$$

$$= 1 + \frac{2}{3} = 1\frac{2}{3}$$

Method II

$$\frac{5}{3} = 5 \div 3$$

$$\begin{array}{r|l} & 1 \\ \hline 3 & 5 \\ & 3 \\ \hline & 2 \end{array}$$

The quotient of 5÷3 is 1 and the remainder is 2.

Let us write the above quotient as the whole number part of the solution. The remainder is the numerator of the fractional part. The denominator is the same as that of the improper fraction.

$$\therefore \frac{5}{3} = 1 + \frac{2}{3} = 1\frac{2}{3}$$

Example 1

Express $\frac{17}{10}$ as a mixed number.

Method I

$$\frac{17}{10} = \frac{10+7}{10}$$
$$= \frac{10}{10} + \frac{7}{10}$$
$$= 1\frac{7}{10}$$

Method II

$$\frac{17}{10} = 17 \div 10 = 1 + \frac{7}{10}$$
$$= 1\frac{7}{10}$$

$$\begin{array}{r|l} & \;\;1 \\ 10 & 17 \\ & 10 \\ \hline & \;\;7 \end{array}$$

Example 2

Express $\frac{17}{4}$ as a mixed number.

Method I

$$\frac{17}{4} = \frac{4+4+4+4+1}{4}$$
$$= \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$$
$$= 1+1+1+1+\frac{1}{4}$$
$$\frac{17}{4} = 4\frac{1}{4}$$

Method II

$$\frac{17}{4} = 17 \div 4 = 4 + \frac{1}{4}$$
$$= 4\frac{1}{4}$$

$$\begin{array}{r|l} & \;\;4 \\ 4 & 17 \\ & 16 \\ \hline & \;\;1 \end{array}$$

Exercise 10.1

(1) Of the fractions given below, choose and write down the improper fractions.

(i) $\frac{8}{6}$, $\frac{49}{50}$, $\frac{31}{30}$, $\frac{19}{3}$, $\frac{3}{4}$

(2) Express each of the following mixed numbers as an improper fraction.

(i) $1\frac{1}{4}$ (ii) $2\frac{3}{5}$ (iii) $3\frac{1}{3}$ (iv) $7\frac{5}{8}$

(3) Express each of the following improper fractions as a mixed number.

(i) $\frac{14}{3}$ (ii) $\frac{13}{5}$ (iii) $\frac{26}{3}$ (iv) $\frac{94}{9}$

(4) Write down as a mixed number and as an improper fraction, the amount of guava that each child receives when 23 equal sized guavas are divided equally among 5 children.

10.5 Comparison of fractions

- **Comparison of fractions having the same numerator**

You have learnt that when two fractions with equal numerators are considered, the fraction with the smaller denominator is greater than the other fraction.

Accordingly, $\frac{4}{5}$ is greater than $\frac{4}{7}$. That is, $\frac{4}{5} > \frac{4}{7}$.

Further, when $\frac{5}{7}, \frac{5}{9}, \frac{5}{8}$ are arranged in ascending order we obtain $\frac{5}{9}, \frac{5}{8}, \frac{5}{7}$. That is, $\frac{5}{9} < \frac{5}{8} < \frac{5}{7}$.

- **Comparison of fractions having the same denominator**

You have also learnt that when two fractions with equal denominators are considered, the fraction with the larger numerator is greater than the other fraction.

Accordingly, $\frac{3}{5}$ is greater than $\frac{2}{5}$. That is, $\frac{2}{5} < \frac{3}{5}$.

Further, when $\frac{9}{11}, \frac{2}{11}, \frac{15}{11}$ are arranged in ascending order we obtain $\frac{2}{11}, \frac{9}{11}, \frac{15}{11}$.

That is, $\frac{2}{11} < \frac{9}{11} < \frac{15}{11}$.

• Further comparison of fractions

Now let us consider how the greater fraction is identified when two fractions with unequal numerators and unequal denominators are compared by writing equivalent fractions having a common denominator.

Let us compare the fractions $\frac{5}{3}$ and $\frac{7}{6}$.

Let us find the fraction with denominator 6 that is equivalent to $\frac{5}{3}$.
To do this, let us multiply the numerator and denominator of $\frac{5}{3}$ by 2.

$$\frac{5}{3} = \frac{5 \times 2}{3 \times 2} = \frac{10}{6}$$

$$\frac{10}{6} > \frac{7}{6}.$$

Since $\frac{10}{6} = \frac{5}{3}$ we obtain $\frac{5}{3} > \frac{7}{6}$.

Therefore, of the two fractions $\frac{5}{3}$ and $\frac{7}{6}$, the larger fraction is $\frac{5}{3}$.

Let us compare the fraction $\frac{7}{12}$ and $\frac{5}{8}$.

Neither denominator of the fractions $\frac{7}{12}$ and $\frac{5}{8}$ can be written as a multiple of the other. In such situations, the fractions have to be converted into equivalent fractions that have a denominator which is a multiple of the denominators of both fractions. It is convenient to take the least common multiple (LCM) of 12 and 8 in this situation.

$$\begin{array}{r|l} 2 & 12,\ 8 \\ \hline 2 & 6,\ 4 \\ \hline & 3,\ 2 \end{array}$$

The least common multiple of 12 and 18 $\Big\} = 2 \times 2 \times 3 \times 2$

$= 24$

$$\frac{7 \times 2}{12 \times 2} = \frac{14}{24}$$

$$\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

$\frac{15}{24} > \frac{14}{24}$ Therefore $\frac{5}{8} > \frac{7}{12}$.

Example 1

Compare the fractions $\frac{17}{12}$ and $\frac{9}{5}$.

There is no number other than 1 which divides both 12 and 5 without remainder.

Therefore, the least common multiple of 12 and 5 $= 12 \times 5 = 60$.

$$\frac{17}{12} = \frac{17 \times 5}{12 \times 5} = \frac{85}{60}$$

$$\frac{9}{5} = \frac{9 \times 12}{5 \times 12} = \frac{108}{60}$$

Since $\frac{108}{60} > \frac{85}{60}$ we obtain, $\frac{9}{5} > \frac{17}{12}$.

A proper fraction is always smaller than an improper fraction with the same denominator.

10.6 Comparison of mixed numbers

- **Mixed numbers with unequal whole number parts**

Let us find the larger number from the mixed numbers $1\frac{1}{2}$ and $3\frac{2}{5}$.

- First, let us examine the whole number parts of the two mixed numbers.
- If the whole number parts are unequal, then the mixed number with the greater whole number part is the larger mixed number.

Accordingly, when the whole number parts of $1\frac{1}{2}$ and $3\frac{2}{5}$ are considered, they are 1 and 3 respectively. Since $3 > 1$, the larger mixed number is $3\frac{2}{5}$.

$3\frac{2}{5} > 1\frac{1}{2}$.

- ## Mixed numbers with equal whole number parts

Select the larger number from $3\frac{2}{5}$ and $3\frac{1}{2}$.

Method I

- The whole number parts of the above two numbers are equal.
- Therefore, let us compare the fractional parts of these mixed numbers.

Accordingly, let us compare the fractional parts $\frac{2}{5}$ and $\frac{1}{2}$ of the mixed numbers $3\frac{2}{5}$ and $3\frac{1}{2}$.

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

Since $\frac{5}{10} > \frac{4}{10}$, we obtain $\frac{1}{2} > \frac{2}{5}$.

Therefore, $3\frac{1}{2} > 3\frac{2}{5}$.

Method II

- Express the mixed numbers as improper fractions.
- The larger mixed number can be selected by considering which equivalent improper fraction is larger.

$$3\frac{2}{5} = \frac{17}{5}$$

$$3\frac{1}{2} = \frac{7}{2}$$

Now let us write $\frac{17}{5}$ and $\frac{7}{2}$ as fractions with equal denominators.

$$\frac{17}{5} = \frac{17 \times 2}{5 \times 2} = \frac{34}{10}$$

$$\frac{7}{2} = \frac{7 \times 5}{2 \times 5} = \frac{35}{10}$$

Since $\frac{35}{10} > \frac{34}{10}$, we obtain $\frac{7}{2} > \frac{17}{5}$.

That is, $3\frac{1}{2} > 3\frac{2}{5}$.

Exercise 10.2

(1) For each of the following parts, select and write down the larger/largest fraction from the given fractions.

(i) $\frac{1}{3}, \frac{1}{5}$ (ii) $\frac{13}{7}, \frac{15}{7}$ (iii) $\frac{5}{11}, \frac{8}{11}, \frac{12}{11}$ (iv) $\frac{11}{3}, \frac{11}{7}, \frac{11}{5}$

(v) $\frac{7}{10}, \frac{4}{5}$ (vi) $\frac{3}{2}, \frac{5}{4}$ (vii) $\frac{3}{4}, \frac{2}{3}$ (viii) $\frac{15}{8}, \frac{7}{3}$

(2) For each of the following parts, select and write down the larger number from the given pair of mixed numbers.

(i) $3\frac{1}{4}, 7\frac{2}{3}$ (ii) $6\frac{2}{5}, 4\frac{1}{2}$ (iii) $5\frac{3}{8}, 5\frac{7}{8}$ (iv) $2\frac{4}{5}, 2\frac{4}{7}$

(v) $6\frac{1}{4}, 6\frac{3}{8}$ (vi) $1\frac{3}{4}, 1\frac{2}{3}$ (vii) $7\frac{5}{6}, 7\frac{4}{5}$ (viii) $6\frac{3}{7}, 6\frac{1}{5}$

(3) Fill in the blanks with the suitable symbol from <, > and =.

(i) $\frac{3}{7}$ .... $\frac{3}{5}$ (ii) $\frac{17}{9}$ ..... $\frac{15}{9}$ (iii) $\frac{25}{8}$ .... $\frac{13}{4}$ (iv) $\frac{4}{5}$ .... $\frac{2}{3}$

(v) $2\frac{1}{6}$ ..... $5\frac{1}{3}$ (vi) $7\frac{1}{2}$ ..... $3\frac{4}{5}$ (vii) $2\frac{1}{5}$ ..... $2\frac{2}{10}$

(viii) $4\frac{2}{3}$ ..... $4\frac{1}{2}$ (ix) $7\frac{3}{8}$ ..... $7\frac{1}{3}$

(4) A person divides a 10 acre land he owns into three equal portions and gives each of his sons a portion. He also divides a 15 acre land he owns into four equal portions and gives each of his daughters a portion. Find out whether the portion a son receives is larger or smaller than the portion a daughter receives. .

(5) The depth of the drain cut by three labourers *A*, *B* and *C* during a day are $1\frac{1}{4}$ m, $2\frac{3}{4}$ m and 2 m respectively. Which labourer has cut the drain with the least depth? Explain your answer.

Summary

- Fractions with the numerator greater or equal to the denominator are defined as improper fractions.
- Numbers which consist of a whole number part and a fractional part are defined as mixed numbers.
- Mixed numbers can be compared by first converting them into equivalent improper fractions.

10 Fractions
(Part II)

By studying this lesson you will be able to

- add and subtract fractions.

10.7 Addition of fractions

• Addition of fractions having the same denominator

In Grade 6 you learnt to add proper fractions with equal denominators as well as proper fractions with unequal denominators.

Let us consider the addition of fractions with equal denominators.

$$\frac{2}{8} + \frac{9}{8} = \frac{2+9}{8} = \frac{11}{8}$$

When fractions with equal denominators are added, the denominator of the answer is the same as the denominators of the fractions that are added. The numerator of the answer is the sum of the numerators of the fractions that are added.

The above answer $\frac{11}{8}$ can also be expressed as a mixed number. Then the answer is $1\frac{3}{8}$.

• Addition of fractions with unequal denominators

When fractions with unequal denominators are being added, the given fractions need to be first converted into equivalent fractions with equal denominators, and then added.

Here, it is convenient to convert the given fractions into equivalent fractions that have the least common multiple of the denominators of the given fractions as the denominator.

Find the value of $\frac{7}{10} + \frac{7}{15}$.

The relationship between the denominators of $\frac{7}{10}$ and $\frac{7}{15}$ cannot be identified easily.

In such situations, where the denominators are not related, the given fractions need to be converted into equivalent fractions which have as their denominators a common multiple of the denominators of the given fractions.

Here is it convenient to select the least common multiple of 10 and 15.

$$\begin{array}{r|l} 5 & 10,\ 15 \\ \hline & 2,\ \ 3 \end{array}$$

$\left.\begin{array}{l}\text{The least common multiple}\\ \text{of 10 and 15}\end{array}\right\} = 5 \times 2 \times 3$

$= 30$

$$\frac{7}{10} = \frac{7 \times 3}{10 \times 3} = \frac{21}{30}$$

$$\frac{7}{15} = \frac{7 \times 2}{15 \times 2} = \frac{14}{30}$$

$$\frac{7}{10} + \frac{7}{15} = \frac{21}{30} + \frac{14}{30} = \frac{35}{30} = \frac{7}{6} = 1\frac{1}{6}$$

Example 1

Find the value of $\frac{3}{2} + \frac{3}{8}$.

$$\frac{3}{2} + \frac{3}{8} = \frac{3 \times 4}{2 \times 4} + \frac{3}{8}$$
$$= \frac{12}{8} + \frac{3}{8}$$
$$= \frac{12 + 3}{8}$$
$$= \frac{15}{8}$$
$$= 1\frac{7}{8}$$

Example 2

Find the value of $\frac{1}{4} + \frac{2}{5}$.

Here it is convenient to take the least common multiple of 4 and 5 as the denominator each of the equivalent fractions. The least common multiple of 4 and 5 is 20.

$$\frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20}$$
$$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}$$
$$\frac{1}{4} + \frac{2}{5} = \frac{5}{20} + \frac{8}{20}$$
$$= \frac{13}{20}$$

Example 3

Find the value of $\frac{17}{12} + \frac{9}{8}$.

The least common multiple of 12 and 8 is 24.

$$\frac{17}{12} + \frac{9}{8} = \frac{34}{24} + \frac{27}{24}$$
$$= \frac{61}{24}$$
$$= 2\frac{13}{24}$$

Example 4

Find the value of $\frac{5}{3} + \frac{3}{8} + \frac{7}{4}$.

The least common multiple of 3, 8 and 4 is 24.

$$\frac{5}{3} + \frac{3}{8} + \frac{7}{4} = \frac{40}{24} + \frac{9}{24} + \frac{42}{24}$$
$$= \frac{91}{24}$$
$$= 3\frac{19}{24}$$

- When adding fractions, some of the steps given in the above examples can be done mentally and the answer can be obtained in a few steps.
- When fractions are simplified, if the answer obtained is a proper fraction, it should be given in its simplest form, and if it is an improper fraction, it should be written as a mixed number.

Exercise 10.3

(1) Evaluate the following.

(i) $\frac{2}{9} + \frac{7}{9} + \frac{5}{9}$ (ii) $\frac{13}{11} + \frac{4}{11}$ (iii) $\frac{7}{6} + \frac{13}{12}$ (iv) $\frac{2}{7} + \frac{3}{5}$

(v) $\frac{12}{5} + \frac{1}{3} + \frac{2}{15}$ (vi) $\frac{13}{4} + \frac{2}{5}$ (vii) $\frac{3}{2} + \frac{5}{4} + \frac{4}{3}$

• Addition of mixed numbers

Let us consider how the two mixed numbers $1\frac{2}{5}$ and $1\frac{1}{5}$ are added together. This is written as $1\frac{2}{5} + 1\frac{1}{5}$.

Method I

The whole number parts and the fractional parts can be added separately.

$$1\frac{2}{5} + 1\frac{1}{5} = 1 + 1 + \frac{2}{5} + \frac{1}{5}$$
$$= 2 + \frac{2+1}{5}$$
$$= 2 + \frac{3}{5}$$
$$= 2\frac{3}{5}$$

Method II

The mixed numbers can be written as improper fractions and added.

$1\frac{2}{5} = \frac{7}{5}$ and $1\frac{1}{5} = \frac{6}{5}$

$$1\frac{2}{5} + 1\frac{1}{5} = \frac{7}{5} + \frac{6}{5}$$
$$= \frac{7+6}{5}$$
$$= \frac{13}{5}$$
$$= 2\frac{3}{5}$$

Method I is more suitable.

Example 1

Find the value of $2\frac{3}{7} + \frac{2}{7}$.

$$2\frac{3}{7} + \frac{2}{7} = 2 + \frac{3}{7} + \frac{2}{7}$$
$$= 2 + \frac{5}{7}$$
$$= 2\frac{5}{7}$$

Example 2

Find the value of $1\frac{1}{3} + 2\frac{5}{12}$.

$$1\frac{1}{3} + 2\frac{5}{12} = (1 + 2) + \left(\frac{1}{3} + \frac{5}{12}\right)$$
$$= 3 + \left(\frac{1\times 4}{3\times 4} + \frac{5}{12}\right)$$
$$= 3 + \left(\frac{4}{12} + \frac{5}{12}\right)$$
$$= 3 + \frac{9}{12} = 3\frac{9}{12} = 3\frac{3}{4}$$

Example 3

Find the value of $2\frac{2}{3} + \frac{1}{4}$.

$$2\frac{2}{3} + \frac{1}{4} = 2 + \left(\frac{2}{3} + \frac{1}{4}\right)$$
$$= 2 + \left(\frac{8}{12} + \frac{3}{12}\right)$$
$$= 2 + \left(\frac{8+3}{12}\right)$$
$$= 2 + \frac{11}{12}$$
$$= 2\frac{11}{12}$$

Example 4

Find the value of $2\frac{1}{5} + 4\frac{2}{3}$.

$$2\frac{1}{5} + 4\frac{2}{3} = (2+4) + \left(\frac{1}{5} + \frac{2}{3}\right)$$
$$= 6 + \left(\frac{3}{15} + \frac{10}{15}\right)$$
$$= 6 + \left(\frac{3+10}{15}\right)$$
$$= 6 + \frac{13}{15}$$
$$= 6\frac{13}{15}$$

Example 5

Find the value of $1\frac{2}{3} + 2\frac{3}{5} + \frac{5}{6}$.

$$1\frac{2}{3} + 2\frac{3}{5} + \frac{5}{6} = (1+2) + \left(\frac{2}{3} + \frac{3}{5} + \frac{5}{6}\right)$$
$$= 3 + \left(\frac{20}{30} + \frac{18}{30} + \frac{25}{30}\right) = 3 + \frac{63}{30} = 3 + \frac{63 \div 3}{30 \div 3}$$
$$= 3 + \frac{21}{10}$$
$$= 3 + 2\frac{1}{10}$$
$$= 5\frac{1}{10}$$

Exercise 10.4

(1) Evaluate the following.

(i) $3\frac{2}{7} + \frac{3}{7}$ (ii) $2\frac{4}{10} + 3\frac{3}{10}$ (iii) $1\frac{1}{9} + 2\frac{2}{9} + \frac{4}{9}$

(iv) $2\frac{1}{3} + 3\frac{5}{9}$ (v) $\frac{7}{12} + 2\frac{1}{3}$ (vi) $4\frac{3}{5} + 2\frac{1}{10}$

(vii) $2\frac{1}{4} + \frac{2}{3}$ (viii) $5\frac{2}{3} + 3\frac{2}{5}$ (ix) $2\frac{2}{7} + 1\frac{3}{4}$

(x) $4\frac{3}{10} + 3\frac{1}{4}$ (xi) $5\frac{2}{5} + 2\frac{3}{7}$ (xii) $2\frac{7}{12} + 3\frac{5}{8}$

(xiii) $1\frac{2}{3} + 2\frac{1}{3} + 2\frac{5}{6}$ (xiv) $3\frac{1}{4} + 1\frac{1}{2} + \frac{1}{6}$

(xv) $3\frac{5}{6} + 2\frac{3}{4} + 5\frac{1}{3}$

(2) A seamstress states that $1\frac{1}{6}$ m of material is required for a shirt and $2\frac{3}{4}$ m of material is required for a dress. Find the amount of material of a certain type that is required for a shirt and a dress.

(3) A farmer has cultivated paddy in an area of $3\frac{1}{2}$ square kilometres and vegetables in an area of $1\frac{2}{5}$ square kilometres. Find the total cultivated area.

10.8 Subtraction of fractions

Let us now, using examples, describe how to subtract fractions with equal denominators as well as fractions with unequal denominators.

When fractions with unequal denominators are being subtracted, the given fractions are first converted into equivalent fractions with equal denominators and then subtracted.

Example 1

Find the value of $\frac{7}{5} - \frac{1}{5}$.

$$\frac{7}{5} - \frac{1}{5} = \frac{7-1}{5}$$
$$= \frac{6}{5}$$
$$= 1\frac{1}{5}$$

Example 2

Find the value of $\frac{17}{8} - \frac{3}{2}$.

$$\frac{17}{8} - \frac{3}{2} = \frac{17}{8} - \frac{3}{2}$$
$$= \frac{17}{8} - \frac{12}{8}$$
$$= \frac{17-12}{8}$$
$$= \frac{5}{8}$$

Example 3

Find the value of $\frac{1}{2} - \frac{1}{3}$.

$$\frac{1}{2} = \frac{1\times 3}{2\times 3} = \frac{3}{6}, \qquad \frac{1}{3} = \frac{1\times 2}{3\times 2} = \frac{2}{6}$$

$$\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{3-2}{6} = \frac{1}{6}$$

Exercise 10.5

(1) Find the value of each of the following.

(i) $\frac{8}{11} - \frac{7}{11}$ (ii) $\frac{13}{12} - \frac{7}{12}$ (iii) $\frac{3}{5} - \frac{1}{5}$ (iv) $\frac{19}{11} - \frac{8}{11}$

(v) $\frac{3}{4} - \frac{1}{2}$ (vi) $\frac{2}{3} - \frac{7}{12}$ (vii) $\frac{15}{7} - \frac{11}{14}$ (viii) $\frac{13}{10} - \frac{1}{2}$

(ix) $\frac{3}{2} - \frac{6}{5}$ (x) $\frac{5}{6} - \frac{3}{4}$ (xi) $\frac{11}{7} - \frac{4}{5}$

(xii) $\frac{9}{8} - \frac{5}{6}$ (xiii) $\frac{7}{8} - \frac{5}{12}$ (xiv) $\frac{8}{9} - \frac{5}{6}$

Subtracting mixed numbers

Mother had $3\frac{2}{3}$ m of material. She cut $1\frac{1}{3}$ m from this material to sew a dress for her daughter. The amount of material that is remaining can be written as follows.

Amount of material remaining $= 3\frac{2}{3} - 1\frac{1}{3}$.

Method I

In instances such as this, where mixed numbers are being subtracted, the whole number parts and the fractional parts can be simplified separately. Now let us consider how this is done.

$$3\frac{2}{3} - 1\frac{1}{3} = (3-1) + \frac{2}{3} - \frac{1}{3}$$

$$= 2 + \frac{2-1}{3}$$

$$= 2 + \frac{1}{3}$$

$$= 2\frac{1}{3}$$

Method II

This simplification can also be done by converting the mixed numbers into improper fractions. Let us now consider how this is done.

$$3\frac{2}{3} - 1\frac{1}{3} = \frac{11}{3} - \frac{4}{3}$$

$$= \frac{11-4}{3}$$

$$= \frac{7}{3}$$

$$= 2\frac{1}{3}$$

Example 1

Evaluate $2\frac{7}{9} - \frac{2}{9}$.

$$2\frac{7}{9} - \frac{2}{9} = 2 + \left(\frac{7}{9} - \frac{2}{9}\right)$$
$$= 2 + \left(\frac{7-2}{9}\right)$$
$$= 2 + \frac{5}{9}$$
$$= 2\frac{5}{9}$$

Example 2

Evaluate $6\frac{5}{9} - \frac{1}{3}$.

$$6\frac{5}{9} - \frac{1}{3} = 6 + \left(\frac{5}{9} - \frac{1}{3}\right)$$
$$= 6 + \left(\frac{5}{9} - \frac{1 \times 3}{3 \times 3}\right)$$
$$= 6 + \left(\frac{5}{9} - \frac{3}{9}\right)$$
$$= 6 + \frac{2}{9} = 6\frac{2}{9}$$

Example 3

Evaluate $3\frac{4}{5} - 2\frac{1}{5}$.

$$3\frac{4}{5} - 2\frac{1}{5} = (3-2) + \left(\frac{4}{5} - \frac{1}{5}\right)$$
$$= 1 + \left(\frac{4-1}{5}\right)$$
$$= 1\frac{3}{5}$$

Example 4

Evaluate $5\frac{7}{10} - 2\frac{2}{15}$.

$$5\frac{7}{10} - 2\frac{2}{15} = (5-2) + \left(\frac{7}{10} - \frac{2}{15}\right)$$
$$= 3 + \left(\frac{21}{30} - \frac{4}{30}\right)$$
$$= 3 + \frac{17}{30}$$
$$= 3\frac{17}{30}$$

Example 5

Evaluate $7\frac{2}{3} - \frac{1}{4}$.

$$7\frac{2}{3} - \frac{1}{4} = 7 + \left(\frac{2}{3} - \frac{1}{4}\right)$$

The LCM of 3 and 4 is 12.

$$7\frac{2}{3} - \frac{1}{4} = 7 + \left(\frac{2 \times 4}{3 \times 4} - \frac{1 \times 3}{4 \times 3}\right)$$
$$= 7 + \left(\frac{8}{12} - \frac{3}{12}\right)$$
$$= 7 + \frac{5}{12} = 7\frac{5}{12}$$

Example 6

Evaluate $3\frac{1}{5} - 2\frac{1}{10}$.

$$3\frac{1}{5} - 2\frac{1}{10} = (3-2) + \left(\frac{1}{5} - \frac{1}{10}\right)$$
$$= 1 + \left(\frac{1 \times 2}{5 \times 2} - \frac{1}{10}\right)$$
$$= 1 + \left(\frac{2}{10} - \frac{1}{10}\right)$$
$$= 1 + \frac{1}{10}$$
$$= 1\frac{1}{10}$$

Example 7

Evaluate $3\frac{2}{7} - 1\frac{1}{2}$.

Method I

$$3\frac{2}{7} - 1\frac{1}{2} = (3-1) + \left(\frac{2}{7} - \frac{1}{2}\right)$$
$$= 2 + \left(\frac{4}{14} - \frac{7}{14}\right)$$
$$= 2 + \frac{4-7}{14}$$
$$= 1 + 1 + \frac{4-7}{14}$$
$$= 1 + \frac{14}{14} + \frac{4-7}{14}$$
$$= 1 + \frac{14 + 4 - 7}{14} = 1 + \frac{11}{14}$$
$$= 1\frac{11}{14}$$

Method II

$$3\frac{2}{7} - 1\frac{1}{2} = \frac{23}{7} - \frac{3}{2}$$
$$= \frac{46}{14} - \frac{21}{14}$$
$$= \frac{25}{14}$$
$$= 1\frac{11}{14}$$

In such instances, it is easier to first convert the mixed numbers into improper fractions and then simplify them.

Exercise 10.6

(1) Evaluate the following.

(i) $2\frac{3}{5} - 1\frac{1}{5}$ (ii) $4\frac{5}{7} - 1\frac{4}{7}$ (iii) $2\frac{7}{8} - \frac{4}{8}$

(iv) $2 - 1\frac{1}{4}$ (v) $3 - 1\frac{5}{6}$ (vi) $2 - 1\frac{5}{16}$

(vii) $8\frac{7}{10} - 3\frac{2}{5}$ (viii) $2\frac{2}{5} - 1\frac{3}{20}$ (ix) $2\frac{2}{3} - 1\frac{1}{2}$

(x) $3\frac{3}{4} - 1\frac{7}{18}$ (xi) $6\frac{5}{8} - 4\frac{1}{6}$ (xii) $4\frac{3}{10} - 2\frac{4}{15}$

(2) Sachi travelled $3\frac{7}{10}$ kilometres to his brother Gamini's house by going the initial $3\frac{1}{2}$ kilometres by bus and walking the remaining distance. What was the distance that Sachi walked?

(3) A farmer owns a plot of land of area 4 hectares. He has cultivated kurakkan (finger millet) in $2\frac{1}{2}$ hectares. What is the extent of the land in which he has not cultivated kurakkan?

Miscellaneous Exercise

(1) (i) Express $7\frac{3}{5}$ as an improper fraction.

(ii) Express $\frac{50}{11}$ as a mixed number.

(2) (i) Write the fractions $1\frac{1}{4}, \frac{15}{7}, \frac{5}{3}, \frac{1}{2}$ in ascending order.

(ii) Write the fractions $2\frac{5}{3}, 7\frac{1}{3}, \frac{3}{4}, \frac{5}{7}$ in descending order.

(3) Evaluate the following.

(i) $\frac{1}{5} + 1\frac{1}{4} + 3\frac{5}{7}$ (ii) $\frac{3}{5} + 3\frac{5}{7} + 5\frac{1}{4}$ (iii) $7\frac{2}{3} - 4\frac{1}{4}$

(iv) $4\frac{5}{6} - 1\frac{3}{5}$ (v) $4\frac{5}{8} - 2\frac{1}{3}$ (vi) $2\frac{1}{2} - 1\frac{3}{4}$

(4) Malinga walked for three hours at $3\frac{1}{2}$ kilometres per hour. Find the total distance he walked during the three hours as an improper fraction.

Summary

- When fractions are simplified, if the answer obtained is a proper fraction, it should be given in its simplest form, and if it is an improper fraction, it should be written as a mixed number.

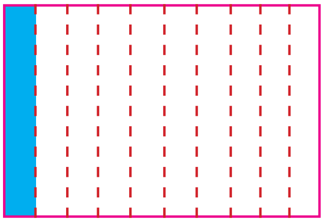
11

Decimals

By studying this lesson you will be able to
- represent a fraction with a denominator that can be written as a power of ten, as a decimal number,
- represent a decimal number as a fraction, and
- multiply and divide a decimal number by a whole number.

11.1 Writing a proper fraction with a denominator which is a power of ten, as a decimal number

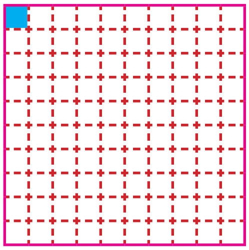
In Grade 6 we learnt how to write a proper fraction with 10 or 100 as the denominator, as a decimal number.



When a unit is divided into 10 equal parts, then one part is equal to $\frac{1}{10}$.

This is denoted as a decimal number, by 0.1.

That is, $0.1 = \frac{1}{10}$.



When a unit is divided into 100 equal parts, then one part is equal to $\frac{1}{100}$.
This is denoted as a decimal number, by 0.01.
That is, $0.01 = \frac{1}{100}$.

We have learnt that when a unit is divided into 1000 equal parts, then one part is equal to $\frac{1}{1000}$.
$\frac{1}{1000}$ is written in decimal form as 0.001. That is, $0.001 = \frac{1}{1000}$.
The number 0.001 is read as zero point zero zero one. The position where 1 is written after the second decimal place in 0.001 is defined as the third decimal place. The place value of the third decimal place is $\frac{1}{1000}$.

Since $\frac{7}{1000}$ is seven $\frac{1}{1000}$ s, we obtain $\frac{7}{1000} = 0.007$ The number 0.007 is read as zero point zero zero seven.

Let us consider $\frac{24}{1000}$.

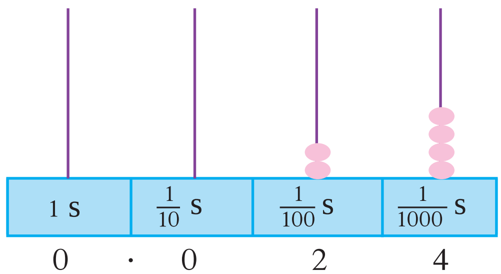
$$\frac{24}{1000} = \frac{20}{1000} + \frac{4}{1000}$$

Since, $\frac{20}{1000} = \frac{20 \div 10}{1000 \div 10} = \frac{2}{100}$

$\frac{24}{1000}$ = two $\frac{1}{100}$ s + four $\frac{1}{1000}$ s.

Accordingly, $\frac{24}{1000} = 0.024$.

0.024 is read as zero point zero two four.
Let us represent 0.024 on an abacus.



Example 1

(1) Write each of the following fractions as a decimal number.

(i) $\frac{4}{1000}$ (ii) $\frac{97}{1000}$ (iii) $\frac{751}{1000}$

(i) $\frac{4}{1000} = 0.004$ (ii) $\frac{97}{1000} = 0.097$ (iii) $\frac{751}{1000} = 0.751$

Exercise 11.1

(1) Express each of the following fractions as a decimal number. Represent them on an abacus.

(i) $\frac{9}{10}$ (ii) $\frac{75}{100}$ (iii) $\frac{9}{1000}$ (iv) $\frac{25}{1000}$ (v) $\frac{275}{1000}$

11.2 Writing a proper fraction with a denominator which is not a power of ten, as a decimal number

Let us learn how to express a proper fraction with a denominator which is not a power of 10, as a decimal number.

- The given fraction can easily be written as a decimal number if it can be converted into an equivalent fraction which has a power of 10 as its denominator.

Let us express $\frac{1}{2}$ as a decimal number.

10 can be divided by 2 without remainder. 10 ÷ 2 = 5. Therefore, by multiplying the numerator and the denominator of $\frac{1}{2}$ by 5, it can be converted into an equivalent fraction with 10 as the denominator.

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

$$\frac{5}{10} = 0.5$$

Therefore, $\frac{1}{2} = 0.5$.

Let us express $\frac{1}{4}$ as a decimal number.

Although 10 cannot be divided by 4 without remainder, 100 can be divided by 4 without remainder. 100 ÷ 4 = 25.

Therefore, by multiplying the numerator and the denominator of $\frac{1}{4}$ by 25, it can be converted into an equivalent fraction with 100 as the denominator.

$$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100}$$

$$\frac{25}{100} = 0.25$$

Therefore, $\frac{1}{4} = 0.25$.

Let us express $\frac{1}{8}$ as a decimal number.

Although 10 and 100 cannot be divided by 8 without remainder, 1000 can be divided by 8 without remainder. 1000 ÷ 8 = 125.

Therefore, by multiplying the numerator and the denominator of $\frac{1}{8}$ by 125, it can be converted into an equivalent fraction with 1000 as the denominator.

$$\frac{1}{8} = \frac{1 \times 125}{8 \times 125} = \frac{125}{1000}$$

$$\frac{125}{1000} = 0.125$$

Therefore, $\frac{1}{8} = 0.125$.

According to the above description, the proper fractions that can be converted into equivalent fractions with a power of 10 as the denominator, can easily be expressed as decimal numbers.

That is, if 10, 100, 1000 or any other power of 10 can be divided without remainder by the denominator of a given fraction, then that fraction can be written as a decimal number with one or more decimal places.

Example 1

Express each of the fractions $\frac{1}{5}$, $\frac{13}{25}$ and $\frac{77}{125}$ as a decimal number.

$\frac{1}{5} = \frac{2}{10} = 0.2$

$\frac{13}{25} = \frac{52}{100} = 0.52$

$\frac{77}{125} = \frac{77 \times 8}{125 \times 8} = \frac{616}{1000} = 0.616$

11.3 Writing a mixed number as a decimal number

Now let us consider how a mixed number is expressed as a decimal number.

Let us write $3\frac{5}{20}$ as a decimal number.

$$3\frac{5}{20} = 3 + \frac{5}{20}$$
$$= 3 + \frac{5 \times 5}{20 \times 5} = 3 + \frac{25}{100}$$
$$= 3 + 0.25$$
$$= 3.25$$

Let us write $7\frac{11}{40}$ as a decimal number.

$$7\frac{11}{40} = 7 + \frac{11}{40}$$
$$= 7 + \frac{11 \times 25}{40 \times 25}$$
$$= 7 + \frac{275}{1000}$$
$$= 7.275$$

11.4 Writing an improper fraction as a decimal number

Let us consider how an improper fraction is written as a decimal number.

Let us write $\frac{17}{5}$ as a decimal number.

Method I

$\frac{17}{5} = 3\frac{2}{5} = 3 + \frac{2}{5}$

$= 3 + \frac{4}{10} = 3 + 0.4$

$= 3.4$

Method II

$\frac{17}{5} = \frac{34}{10} = \frac{30}{10} + \frac{4}{10}$

$= 3 + 0.4$

$= 3.4$

Example 1

Express $\frac{9}{8}$ as a decimal number.

Method I

$\frac{9}{8} = 1 + \frac{1}{8}$

$\frac{9}{8} = 1 + \frac{125}{1000}$

$= 1 + 0.125$

$= 1.125$

Method II

$\frac{9}{8} = \frac{9 \times 125}{8 \times 125}$

$= \frac{1125}{1000} = \frac{1000}{1000} + \frac{125}{1000}$

$= 1 + 0.125$

$= 1.125$

Exercise 11.2

Express the following fractions and mixed numbers as decimal numbers.

(i) $\frac{3}{5}$ (ii) $\frac{3}{4}$ (iii) $\frac{8}{25}$ (iv) $\frac{321}{500}$ (v) $\frac{39}{40}$

(vi) $13\frac{1}{2}$ (vii) $2\frac{7}{50}$ (viii) $2\frac{1}{8}$ (ix) $3\frac{7}{40}$ (x) $5\frac{14}{125}$

(xi) $\frac{13}{10}$ (xii) $\frac{27}{20}$ (xiii) $\frac{7}{5}$ (xiv) $\frac{97}{8}$ (xv) $\frac{251}{250}$

11.5 Writing a decimal number as a fraction

Let us write 0.5 as a fraction.

$0.5 = \frac{5}{10}$

To express $\frac{5}{10}$ in its simplest form, let us divide the numerator and the denominator by 5.

$0.5 = \frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$

Let us write 0.375 as a fraction.

There fore, $0.375 = \frac{375}{1000}$

To express $\frac{375}{1000}$ in its simplest form, let us divide the numerator and the denominator by 125.

$\frac{375}{1000} = \frac{375 \div 125}{1000 \div 125} = \frac{3}{8}$

$0.375 = \frac{3}{8}$

Let us write 1.75 as a fraction.

$1.75 = 1 + 0.75 = 1 + \frac{75}{100} = 1\frac{75}{100}$

To express $\frac{75}{100}$ in its simplest form, let us divide the numerator and the denominator by 25.

$\frac{75}{100} = \frac{75 \div 25}{100 \div 25} = \frac{3}{4}$

Therefore, $1.75 = 1\frac{3}{4}$.

Example 1

Express 1.625 as a fraction in its simplest form.

$$1.625 = 1 + 0.625 = 1 + \frac{625}{1000} = 1 + \frac{625 \div 25}{1000 \div 25} = 1 + \frac{25}{40} = 1 + \frac{25 \div 5}{40 \div 5}$$
$$= 1 + \frac{5}{8}$$
$$= 1\frac{5}{8}$$

Exercise 11.3

Write each of the following decimal numbers as a fraction and express it in the simplest form.

(i) 0.7	(ii) 1.3	(iii) 0.45	(iv) 8.16
(v) 6.75	(vi) 0.025	(vii) 4.225	(viii) 8.625

11.6 Multiplying a decimal number by a whole number

$2 \times 3 = 2 + 2 + 2 = 6$

This illustrates the fact that the product of two whole numbers can be obtained by writing it as a sum.

Now let us find the value of 0.1×3.

$0.1 \times 3 = 0.1 + 0.1 + 0.1$

$= 0.3$

Let us find the value of 0.8×2.

$0.8 \times 2 = 0.8 + 0.8$

$= 1.6$

Let us find the value of 0.35×4.

$0.35 \times 4 = 0.35 + 0.35 + 0.35 + 0.35$

$= 1.40$

$= 1.4$

Let us examine the above answers by considering the following table.

$0.1 \times 3 = 0.3$	$1 \times 3 = 3$
$0.8 \times 2 = 1.6$	$8 \times 2 = 16$
$0.35 \times 4 = 1.40$	$35 \times 4 = 140$

It will be clear to you from observing the above table that, when multiplying a decimal number by a whole number, the answer can be obtained by following the steps given below too.

- Consider the decimal number as a whole number by disregarding the decimal point and multiply it by the given whole number.

- Place the decimal point in the answer that is obtained such that the final answer has the same number of decimal places as the original decimal number.

Now let us find the value of 24.31×6.
First let us multiply the numbers without taking the decimal places into consideration.

$$\begin{array}{r} 2431 \\ \times \quad 6 \\ \hline 14586 \\ \hline\hline \end{array}$$

Since 24.31 has two decimal places, place the decimal point such that the final answer too has two decimal places. Then $24.31 \times 6 = 145.86$

It must be clear to you that, when the whole number by which the decimal number has to be multiplied is large, the method given above is much easier to use than the method of repeatedly adding the decimal number.

Example 1

Find the value of 4.276×12.

$$\begin{array}{r} 4276 \\ \times \quad 12 \\ \hline 8552 \\ 4276 \\ \hline 51312 \\ \hline\hline \end{array}$$

Since 4.276 has three decimal places, the decimal point is placed such that the answer too has three decimal places.

Then, $4.276 \times 12 = 51.312$

Exercise 11.4

Evaluate the following.

(i) 2.45×6	(ii) 0.75×4	(iii) 3.47×15
(iv) 15.28×13	(v) 0.055×3	(vi) 1.357×41

• Multiplying a decimal number by 10, 100 and 1000

Let us consider the following products.

$2.1 \times 10 = 21.0$	$2.1 \times 100 = 210.0$	$2.1 \times 1000 = 2100.0$
$3.75 \times 10 = 37.50$	$3.75 \times 100 = 375.00$	$3.75 \times 1000 = 3750.00$
$23.65 \times 10 = 236.50$	$23.65 \times 100 = 2365.00$	$23.65 \times 1000 = 23650.00$
$43.615 \times 10 = 436.150$	$43.615 \times 100 = 4361.500$	$43.615 \times 1000 = 43615.000$

The following facts are discovered by examining the above products.

- The number that is obtained when a decimal number is multiplied by 10 can be obtained by moving the decimal point in the original decimal number by one place to the right. $37.16 \times 10 = 371.6$
- The number that is obtained when a decimal number is multiplied by 100 can be obtained by moving the decimal point in the original decimal number by two places to the right. $37.16 \times 100 = 3716$
- The number that is obtained when a decimal number is multiplied by 1000 can be obtained by moving the decimal point in the original decimal number by three places to the right. $37.160 \times 1000 = 37\ 160$

Exercise 11.5

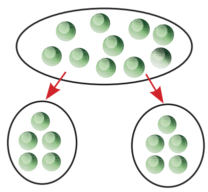
Evaluate the following.

(i) 4.74×10	(ii) 0.503×10	(iii) 0.079×10
(iv) 5.83×100	(v) 5.379×100	(vi) 0.07×100
(vii) 1.2×100	(viii) 0.0056×10	(ix) 0.0307×100
(x) 3.7×1000	(xi) 8.0732×1000	(xii) 6.0051×1000

11.7 Dividing a decimal number by 10, 100 and 1000

$10 = 5 \times 2$ means that there are 2 heaps of five in 10. Therefore, when 10 is divided into two equal heaps there are 5 in each heap.

That is $10 \div 2 = 5$.
You have learnt this in Grade 6.



Now let us find the value of 32.6 ÷ 10.

32.6 ÷ 10 is how many 10s there are in 32.6.
We know that 3.26 × 10 = 32.6.
Therefore, 32.6 ÷ 10 = 3.26

Similarly,
145.56 ÷ 100 is how many 100s there are in 145.56.
Since 1.4556 × 100 = 145.56,
we obtain 145.56 ÷ 100 = 1.4556

6127.3 ÷ 1000 is how many 1000s there are in 6127.3.
Since 6.1273 × 1000 = 6127.3,
we obtain 6127.3 ÷ 1000 = 6.1273.

Let us consider the following divisions.

7871.8 ÷ 10 = 787.18	7871.8 ÷ 100 = 78.718	7871.8 ÷ 1000 = 7.8718
169.51 ÷ 10 = 16.951	169.51 ÷ 100 = 1.6951	169.51 ÷ 1000 = 0.16951
9.51 ÷ 10 = 0.951	9.51 ÷ 100 = 0.0951	9.51 ÷ 1000 = 0.00951

Accordingly,

- The number that is obtained by dividing a decimal number by 10 is equal to the number that is obtained by moving the decimal point in the original decimal number by one decimal place to the left.
  6.0 ÷ 10 = 0.60
- The number that is obtained by dividing a decimal number by 100 is equal to the number that is obtained by moving the decimal point in the original decimal number by two decimal places to the left.
  006.0 ÷ 100 = 0.060 = 0.06
- The number that is obtained by dividing a decimal number by 1000 is equal to the number that is obtained by moving the decimal point in the original decimal number by three decimal places to the left.
  006.0 ÷ 1000 = 0.0060 = 0.006

Exercise 11.6

Evaluate the following.

(i) $27.1 \div 10$ (ii) $1.36 \div 10$ (iii) $0.26 \div 10$ (iv) $0.037 \div 10$
(v) $0.0059 \div 10$ (vi) $58.9 \div 100$ (vii) $3.7 \div 100$ (viii) $97.6 \div 100$
(ix) $0.075 \div 100$ (x) $0.0032 \div 100$ (xi) $4375.8 \div 1000$
(xii) $356.8 \div 1000$

• Dividing a decimal number by a whole number

Let us find the value of $7.5 \div 3$.

Divide the whole number part.
When long division is being performed, place the decimal point in the answer, when the number immediately to the right of the decimal point is being divided. Then continue with the division.

Step 1

$$\begin{array}{r|l} & 2 \\ \hline 3 & 7\,.\,5 \\ & 6 \\ \hline & 1 \end{array} \qquad \begin{array}{l} 2 \times 3 = 6 \\ 7 - 6 = 1 \end{array}$$

$7 \div 3 = 2$ with a remainder of 1

Since the decimal part of 7.5 occurs after 7, place the decimal point after 2 in the answer.

Step 2

$$\begin{array}{r|l} & 2\,.\downarrow \\ \hline 3 & 7\,.\,5 \\ & 6 \\ \hline & 1\;\;5 \end{array}$$

Bring 5 down

Step 3

$$\begin{array}{r|l} & 2\,.\,5 \\ \hline 3 & 7\,.\,5 \\ & 6 \\ \hline & 1\;\;5 \\ & 1\;\;5 \\ \hline & \;\;\;\;0 \end{array} \qquad \begin{array}{l} 5 \times 3 = 15 \\ 15 - 15 = 0 \end{array}$$

Example 1

(i) Find the value of 182.35 ÷ 7.

```
     26.05
 7 ) 182.35
     14
      42
      42
       03
       00
        35
        35
         0
```

Place the decimal point, when the number 3, which is immediately to the right of the decimal place is being divided.

(ii) Find the value of 0.672 ÷ 12.

```
       0.056
 12 ) 0.672
      0
      0 6
        0
        67
        60
         72
         72
          0
```

$0.672 \div 12 = 0.056$

(iii) Find the value of 2.13 ÷ 4.

```
      0.5325
 4 ) 2.1300
     0
     2 1
     2 0
       1 3
       1 2
         10
          8
          20
          20
```

$2.13 \div 4 = 0.5325$

Further knowledge

```
     2 . 5
 3 ) 7 . 5
     6
     1  5
     1  5
        0
```

The digit in the ones place of 7.5 is 7. This denotes 7 ones. When 7 is divided by 3, we obtain 2 and a remainder of 1.

A remainder of one means 1 ones. That is, ten $\frac{1}{10}$ s.

The digit 5 in 7.5 denotes five $\frac{1}{10}$ s. Therefore, there are fifteen $\frac{1}{10}$ s in the first decimal place. Let us divide this fifteen $\frac{1}{10}$ s by 3. Then we obtain five $\frac{1}{10}$ s with no remainder. That is $7.5 \div 3 = 2.5$

Exercise 11.7

(1) Evaluate the following.

(i) $84.6 \div 2$ (ii) $167.2 \div 4$ (iii) $54.6 \div 3$

(iv) $98.58 \div 6$ (v) $74.5 \div 5$ (vi) $35.86 \div 2$

(vii) $0.684 \div 6$ (viii) $0.735 \div 7$ (ix) $1.08 \div 4$

(x) $7.401 \div 3$ (xi) $8.04 \div 8$ (xii) $11.745 \div 9$

(2) If the height of a child is 145 cm, express this height in metres.

Summary

- When multiplying a decimal number by a whole number, consider the decimal number as a whole number by disregarding the decimal point and multiply the two numbers. Place the decimal point in the answer that is obtained so that it has the same number of decimal places as the original decimal number.

- When a decimal number is multiplied by a power of ten, the number of places the decimal point in the decimal number shifts to the right is equal to the number of zeros in the power of ten by which it is multiplied.

- When a decimal number is divided by a power of ten, the number of places the decimal point in the decimal number shifts to the left is equal to the number of zeros in the power of ten by which it is divided.

12 Algebraic Expressions

By studying this lesson you will be able to

- construct algebraic expressions,
- simplify algebraic expressions, and
- find the value of algebraic expressions by substituting numbers.

12.1 Constructing algebraic expressions

Kavin buys the same amount of milk every day. If this amount is not known, then we cannot represent it by a number although it is a constant value.

As in the above situation, when the numerical value of a constant amount is not known, it is defined as an "**unknown constant**".

The daily income of a certain shop takes different values depending on its daily sales. Since the daily income is not a fixed value, it is a **variable**.

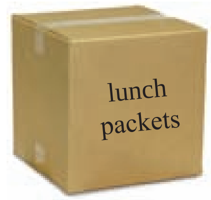
Simple letters of the English alphabet such as $a,b,c,\ldots,x,y,z$ are used to represent unknown constants and variables.

Accordingly, considering the above two examples, the amount of milk bought each day can be denoted by the letter a and the daily income of the shop can be denoted by x.

Let us denote the number of bananas in a bunch in a shop by a. When a comb of 12 bananas is sold, the number of bananas remaining in the bunch can be denoted by $a - 12$.

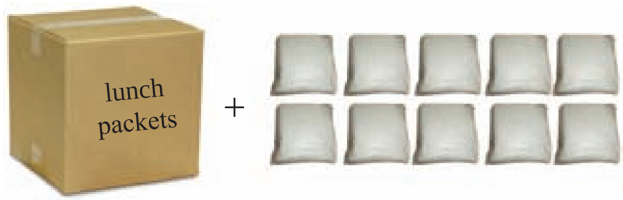
The expression $a - 12$ is an **algebraic expression**. a and 12 are defined as the "**terms**" of this expression.

If the price of a banana is 8 rupees, then $8 \times a$ rupees can be gained by selling all the bananas in the bunch. This is written as $8a$. The coefficient of a in the term $8a$ is 8. There is only one algebraic term in the expression $8a$.



Let us take the number of rice packets sold daily by a vendor as x. If the price of a rice packet is 80 rupees, then the vendor's daily income is $80 \times x$ rupees. We write this as $80x$ rupees.

If the vendor receives a new order to supply 10 more packets daily, then the number of rice packets sold daily will be $x + 10$.



The terms in the expression $x + 10$, are x and 10.

Example 1

The letter m represents a number of unknown value.

(i) Write in terms of m, the number which is three times the given number.

(ii) Write in terms of m, the number which is 15 more than twice the given number.

(i) The number which is three times as large as m is $3m$.

(ii) The number which is twice as large as m is $2\,m$.

Therefore, the number that is greater than $2\,m$ by 15 is $2\,m + 15$.

Exercise 12.1

(1) (i) Construct an algebraic expression for the price of 5 apples by taking the price of one apple as a rupees.

(ii) The price of a pineapple is 10 rupees more than the price of 5 apples. Construct an algebraic expression for the price of a pineapple in terms of a.

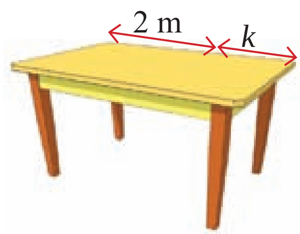
(2) A shop owner buys 12 loaves of bread from a bakery at b rupees per loaf. He then sells these loaves so that each loaf brings him a profit of 3 rupees.

(i) What is the total amount the shop owner pays for the loaves of bread?

(ii) What is the selling price of a loaf of bread?

(iii) A customer buys a loaf of bread and 500g of sugar. The price of 1 kilogram of sugar is 80 rupees. What is the total amount the customer spends?

(3) 1 m = 100 cm.



(i) The length of a table is k centimeters more than 2 meters. Express the length of the table in centimeters in terms of k.

(ii) The width of this table is 50 cm less than its length. Write the width as an expression of k.

12.2 More on constructing algebraic expressions

The algebraic expressions we have constructed so far contain one algebraic symbol, one or more mathematical operations and numbers.

The following table describes algebraic expressions containing one unknown term.

Algebraic Expression	Unknown constant or variable in the algebraic expression	Coefficient of the unknown constant or variable	Terms of the algebraic expression	Mathematical operations in the order they appear in the algebraic expression
$4x$	x	4	$4x$	$\times$
$y+4$	y	1	y , 4	$+$
$p-10$	p	1	p , 10	$-$
$20+3m$	m	3	20 , $3m$	$+$, $\times$
$3a+5$	a	3	$3a$, 5	$\times$, $+$

The mathematical operations of addition, subtraction and multiplication are used in the above expressions. The coefficient of the unknown in each of the expressions is a positive whole number. The operation division is not used in any of these algebraic expressions.

Let us now consider algebraic expressions that have a fraction as the coefficient of the unknown.

There are x number of marbles in a bottle. They are placed in three containers such that each container has the same number of marbles. Then the number of marbles in one container is $x \div 3$. That is, $\frac{x}{3}$.

The width of a hostel room is half its length. If the length is l meters, let us write the width in meters.

The width of the room is $l \div 2$ meters. That is, the width of the room is $\frac{l}{2}$ m.

The length of the adjoining room is one meter more than the width of this room. Let us write the length of the adjoining room as an algebraic expression.

The length of the adjoining room = $\frac{l}{2} + 1$ meters.

Example 1

(1) If more than one meter of cloth is bought, then the price of one meter is p rupees. If less than one meter of cloth is bought, then an additional 10 rupees is charged. Write the price of $\frac{1}{2}$ a meter of cloth as an algebraic expression.

Price of 1m of cloth = p rupees

Since the quantity which is bought is less than 1 m,
the price of $\frac{1}{2}$ m of cloth $= \frac{p}{2} + 10$ rupees.

Example 2

(1) A father sells the 3 plots of land he owns at p rupees per plot. He then divides the money he receives equally among his four children. Write the amount of money received by each child as an algebraic expression.

The money obtained by selling the three plots of land = $3p$ rupees
The amount of money received by each child $= \frac{3p}{4}$ rupees

Exercise 12.2

(1) Complete the following table.

Algebraic expression	Unknown constant or variable in the algebraic expression	Terms of the algebraic expression
$\frac{a}{2} + 5$	a	$\frac{a}{2}, 5$
$\frac{p}{4} - 8$		
$\frac{x}{5} + 10$		
$25 + \frac{y}{3}$		

(2) Construct an algebraic expression for each of the following situations.

(i) The value of a number is denoted by a. What is the value of the number that is greater by 4 than half the value of the given number?

(ii) In a restaurant, a loaf of bread is sold for p rupees. A person buys $\frac{1}{4}$ of a loaf of bread and a dish of dhal. The dish of dhal costs 30 rupees. Write an algebraic expression for the total amount of money the person has to pay.

(iii) The height of a building is 5 meters less than $\frac{1}{2}$ of its length. If its length is l meters, write the height as an expression of l.

(iv) The price of 1kg of sugar is y rupees. If a 100 rupee note is tendered when $\frac{1}{2}$ kg of sugar is bought, write the balance as an algebraic expression of y.

(3) The price of a box of pencils containing 12 pencils is x rupees.

(i) Write the price of one pencil as an algebraic expression.

(ii) If the price of an eraser is 10 rupees, write the amount of money required to buy 2 pencils and an eraser as an algebraic expression.

(4) Write the expressions given below in words.

The expression $5a - 8$ can be expressed in words as follows.

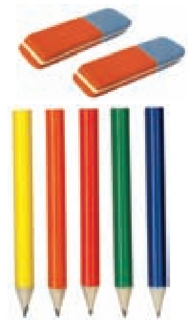
If a denotes a given value, then $5a - 8$ denotes the value which is 8 less than the value of five times a.

(i) $2a + 8$	(ii) $3x - 15$	(iii) $2(p + 5)$
(iv) $\frac{p}{4} - 4$	(v) $20 - 5p$	(vi) $\frac{x}{2} + 14$
(vii) $\frac{y}{5} - 1$	(viii) $30 + \frac{p}{2}$	(ix) $45 - \frac{y}{3}$

12.3 Constructing algebraic expressions having two unknown terms

The price of a pencil is x rupees and the price of an eraser is y rupees. Let us write the price of 5 pencils and 2 erasers as an algebraic expression.

The price of 5 pencils $= 5 \times x$ rupees $= 5x$ rupees
The price of 2 erasers $= 2 \times y$ rupees $= 2y$ rupees
The price of 5 pencils and 2 erasers $= (5x + 2y)$ rupees



The price of 1 kg of sugar is x rupees, the price of 1kg of wheat flour is y rupees and the price of a box of matches is 3 rupees. Let us write the amount of money required to buy 500 g of sugar, 2 kg of wheat flour and 3 boxes of matches as an algebraic expression.

$$\left.\begin{array}{r}\text{The price of 500 g of sugar,}\\ \text{when the price of 1 kg of sugar is } x \text{ rupees}\end{array}\right\} = \frac{x}{2} \text{ rupees}$$

$$\left.\begin{array}{r}\text{The price of 2 kg of wheat flour}\\ \text{when the price of 1 kg of wheat flour is } y \text{ rupees}\end{array}\right\} = 2y \text{ rupees}$$

$$\left.\begin{array}{r}\text{The price of 3 boxes of matches,}\\ \text{when the price of one box of matches is 3 rupees}\end{array}\right\} = 9 \text{ rupees}$$

Therefore, the required amount of money $= \left(\frac{x}{2} + 2y + 9\right)$ rupees

Example 1

(i) There are a number of boys and b number of girls in a class. Write the total number of students in the class as an algebraic expression.
The total number of students in the class $= a + b$

(ii) Write the algebraic expression $\frac{x}{2} + \frac{y}{2}$ in words.
"Add one half of the value represented by y to one half of the value represented by x"

Example 2

25 coconuts were bought at a rupees each and all 25 fruits were sold at b rupees each. Assume that b is greater than a. Write an algebraic expression for the profit.

The price of a coconut = a rupees

The amount of money spent on buying 25 coconuts = $25a$ rupees

The amount of money gained by selling 25 coconuts = $25b$ rupees

Profit $= (25b - 25a)$ rupees

Exercise 12.3

(1) Construct algebraic expressions for the following.

(i) A number is represented by a. What is the number that is greater than a by b?

(ii) A number is represented by p. Write the number that is less than p by q.

(iii) The price of a coconut is x rupees.
The price of 1kg of rice is y rupees.
Write an expression in terms of x and y for the price of 4 coconuts and 3 kg of rice.

(iv) The price of 1 kg of sugar is x rupees and the price of a 250 g packet of tea is y rupees. Find the amount of money required to buy 2 kg and 500 g of sugar and 2 packets of tea.

(v) 250 g = $\frac{1}{4}$ kg. 1kg of potatoes is x rupees. A bundle of green leaves is y rupees. Write an algebraic expression for the amount paid if 250 g of potatoes and a bundle of green leaves are bought.

(vi) There are x number of Sinhala books and y number of English books in the school library. $\frac{1}{2}$ the Sinhala books and $\frac{1}{2}$ the English books are Literature books. If the library has issued 23 Sinhala Literature books and 18 English Literature books, then express the number of literature books remaining in the library as an algebraic expression.

(2) Write the following expressions in words.

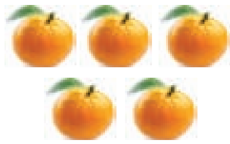
(i) $3x + 5y$ (ii) $2a - 7b$ (iii) $\frac{x}{4} - y + 5$ (iv) $2k + 3p - 8$

12.4 Simplifying the terms of an algebraic expression

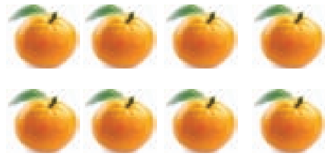
Let us consider an algebraic expression similar to one we constructed earlier.

The price of an orange is a rupees. Nimal bought 5 oranges and Deepani bought 8.

Nimal



Deepani



Nimal spent $5a$ rupees and Deepani spent $8a$ rupees. So the total amount of money spent by both of them is $5a + 8a$.

Since the number of oranges bought by both of them is 13, the total amount spent is $13 \times a$ rupees. That is $13a$ rupees.

This shows that $5a + 8a = 13a$.

Algebraic terms such as $5a$ and $8a$ which have the same unknown are called "**like terms**". By adding or subtracting several such terms, we can simplify them to one term.

There are no like terms in the algebraic expression $4x + 3y + 5$. Such an expression cannot be simplified further. The terms $4x$, $3y$, 5 of this expression are called "**unlike terms**".

Let us simplify $4x + 3y + x + 2y$.

Let us write the like terms together.

$$\begin{aligned} 4x + 3y + x + 2y &= 4x + 1x + 3y + 2y \\ &= 5x + 5y \end{aligned}$$

Let us simplify $10p + 4k + p - k$.

$$\begin{aligned} 10p + 4k + p - k &= 10p + 1p + 4k - 1k \\ &= 11p + 3k \end{aligned}$$

Example 1

Simplify the following.

(i) $3x + 6k + 5x + 3k + 7$ (ii) $5a + b + 8 + 3a - b - 5$

(i) $3x + 6k + 5x + 3k + 7 = 3x + 5x + 6k + 3k + 7$
$= 8x + 9k + 7$

(ii) $5a + b + 8 + 3a - b - 5 = 5a + 3a + b - b + 8 - 5$
$= 8a + 0 + 3$
$= 8a + 3$

Example 2

There are 25 boys and 15 girls in a Grade 4 class.
There are 28 boys and 11 girls in a Grade 5 class.
The price of a pen is p rupees and the price of an eraser is q rupees. Find the total amount of money needed to give a pen to each boy in Grade 4, an eraser to each girl in Grade 4, an eraser to each boy in Grade 5 and a pen to each girl in Grade 5.

The money needed to give pens and erasers to the students in Grade 4 $= 25p + 15q$

The money needed to give pens and erasers to the students in Grade 5 $= 11p + 28q$

The money needed to give pens and erasers to the students in Grade 4 and Grade 5 $= 25p + 15q + 11p + 28q$

$= 25p + 11p + 15q + 28q$
$= 36p + 43q$

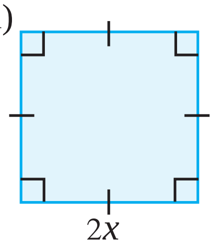
Exercise 12.4

(1) Simplify the following.

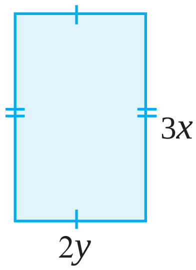
(i) $4x + 5y + 3x + 7$ (ii) $3a + 4 + 6b + 3$
(iii) $5p + 4q - 2p + q$ (iv) $10m - 7n + 10n - 4m$
(v) $3k + 5l + 10 + k + 4l - 5$ (vi) $8x - 4y - 11 + x + 7y + 13$

(2) Write an algebraic expression for the perimeter of each of the figures below. Simplify the expression.

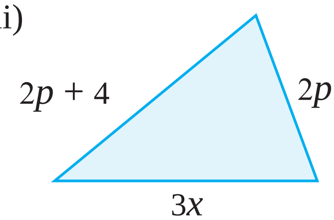
(i)



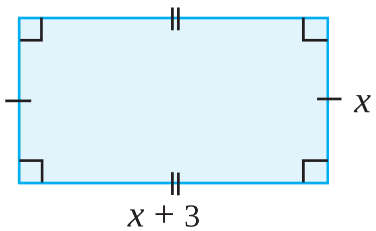
(ii)



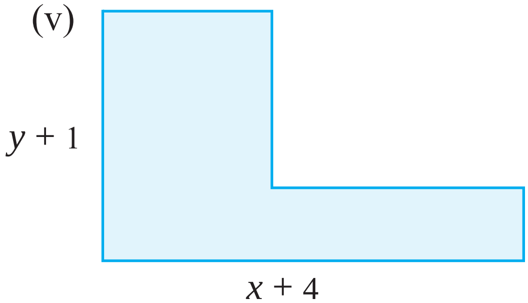
(iii)



(iv)



(v)



12.5 Substituting values for the unknowns in an algebraic expression

When $x = 2$, the expression $x + 3$ takes the value 5. You have learnt in grade 6 that giving a numerical value to the unknown term in an algebraic expression in this manner is called substitution. By substitution, an algebraic expression gets a value.

Let us consider the expression $x + 3$.
When $x = 2$,
$x + 3 = 2 + 3 = 5$.

Let us find the value of $3x - 5$ when $x = 4$.

$$\begin{aligned} 3x - 5 &= 3 \times 4 - 5 \\ &= 12 - 5 = 7 \end{aligned}$$

Let us find the value of $4a - 3$ when $a = 2$.

$$\begin{aligned} 4a - 3 &= 4 \times 2 - 3 \\ &= 8 - 3 \\ &= 5 \end{aligned}$$

Let us now substitute values for the unknowns in an algebraic expression which has two unknown terms and find its value.

Let us find the value of $3x + 4y$ when $x = 4$ and $y = 5$.

$$\begin{aligned} 3x + 4y &= 3 \times 4 + 4 \times 5 \\ &= 12 + 20 \\ &= 32 \end{aligned}$$

Example 1

Find the value of each of the algebraic expressions given below when $x = 4$ and $y = 2$.

(i) $x - y$

$x - y = 4 - 2 = 2$

(ii) $3x - y - 5$

$$\begin{aligned} 3x - y - 5 &= 3 \times 4 - 2 - 5 \\ &= 12 - 2 - 5 \\ &= 10 - 5 \\ &= 5 \end{aligned}$$

Exercise 12.5

(1) Find the value of each of the algebraic expressions given below when $a = 4$.

(i) $3a - 5$ (ii) $5\,(a - 3)$ (iii) $15 - 2a$ (iv) $7a - 5$

(2) For each of the values given to x, find the value of $6x + 4$.

(i) $x = 1$ (ii) $x = 2$ (iii) $x = 5$ (iii) $x = 12$

(3) Find the value of each of the given expressions by substituting the given values.

(i) $4x - 13y + 5$ when $x = 4$ and $y = 1$

(ii) $7a$ - $3b$ $- 8$, when $a = 3$ and $b = 1$

(iii) $2p + k - 5$, when $p = 6$ and $k = 2$

Miscellaneous Exercise

(1) The length of a room is x meters less than twice its width. The width of the room is 3 m. Write an expression in terms of x for the length of the room.

(2) The price of a pen is x rupees and the price of 12 books is y rupees. Nimal buys 2 pens and 3 books. Write an expression for the total amount of money spent by Nimal.

(3) Write each expression given below in words.

(i) $8 + \frac{y}{2}$ (ii) $16 - \frac{a}{3}$

(4) Simplify the following.

(i) $8a + 7b - 3 - 6b - 2a$ (ii) $6x + 5y - 6x - 3y$

(5) Find the value of each of the expressions given below when $x = 7$ and $y = 3$.

(i) $6x - 5y$ (ii) $7x - 3 - 6y$

(6) A father's age was 35 years at the time his son was born.

(i) Write the age of the father, when his son is x years old.

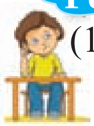
(ii) The mother is 4 years younger to the father. Write the mother's age in terms of x when the son is x years old.

(iii) How many years older is the mother than the son?

Summary

- In an algebraic expression, the number written together with an unknown is called the "coefficient of the unknown".
- Algebraic terms with the same unknown are called "like terms".
- Several like terms can be simplified into one term by adding or subtracting them.
- Algebraic terms with different unknowns are called "unlike terms".
- Two unlike terms cannot be simplified further into one term by adding or subtracting them.

Ponder



(1) A vendor sells 1kg of brinjals for 10 rupees more than twice the price he paid for 1 kg of brinjals. He sells 1kg of papaw for 8 rupees more than three times the price he paid for 1 kg of papaw.

The vendor buys 1kilogram of brinjals and 1kilogram of papaw for x rupees and y rupees respectively.

(i) Write an algebraic expression for the amount the vendor spent to buy 1 kg of brinjals and 1 kg of papaw.

(ii) Write an algebraic expression for the selling price of 1 kg of brinjals.

(iii) Write an algebraic expression for the selling price of 1 kg of papaw.

(iv) Write an algebraic expression for the amount he receives by selling 1 kg of brinjals and 1 kg of papaw.

(v) If the vendor bought 1 kg of brinjals for 35 rupees and 1 kg of papaw for 20 rupees, obtain values for the algebraic expressions in (i), (ii), (iii) and (iv) above.

Glossary

Acute angle	සුළු කෝණය	கூர்ங்கோணம்
Addition	එකතු කිරීම	கூட்டல்ஸ
Algebraic expressions	වීජීය ප්‍රකාශන	அட்சரகணிதக் கோவை
Algebraic symbols	වීජීය සංකේත	அட்சரக் குறியீடு
Algebraic terms	වීජීය පද	அட்சரகணித உறுப்பு
Angle	කෝණය	கோணம்
Axis of symmetry	සමමිති අක්ෂය	சமச்சீர் அச்சு
Bilateral symmetry	ද්විපාර්ශ්වික සමමිතිය	இருபுடைச் சமச்சீர
Century	ශතකය	நூற்றாண்டு
Coefficient	සංගුණක	வகுத்தல்
Decade	දශකය	தசாப்தம
Decimal numbers	දශම සංඛ්‍යා	முடிவுள்ள தசமம்
Denominator	හරය	பகுதி
Digital root	ඉලක්කම් දර්ශකය	இலக்கச்சுட்டி
Directed number	සදිශ සංඛ්‍යාව	திசைகொண்ட எண்கள்
Division	බෙදීම	வகுத்தல்
Dynamic concept	ගතික සංකල්පය	கோணங்களைப்
Elements	අවයව	மூலகங்கள
Equivalent fraction	තුල්‍ය භාගය	சமவலுப் பின்னங்கள்
Expansion of powers	බල ප්‍රසාරණය	வலுக்களின் விரிவு
Factor	සාධකය	காரணி
Fraction	භාගය	பின்னம்
Greatest common factor	මහා පොදු සාධකය	பொதுக்காரணிகளுள் பெரியது்
Improper fraction	විෂම භාගය	முறைமை இல்லாப் பின்னம
Integers	නිඛිල	நிறைவெண்
Leap year	අධික අවුරුද්ද	நெட்டாண்டு
Least common multiple	කුඩා ම පොදු ගුණාකාරය	பொது மடங்குகளுள்
Like terms	සජාතීය පද	நிகர்த்த உறுப்புக்கள
Linear algebraic expressions	ඒකජ වීජීය ප්‍රකාශන	ஒருறுப்பு அட்சரகணிதக்கோவை

Mass	ස්කන්ධය	திணிவு
Mathematical operations	ගණිත කර්ම	கணிதச் செய்கைகள
Millennium	සහස්‍රකය	ஆயிரம் ஆண்டு
Minus	ඍණ	மறை
Mixed number	මිශ්‍ර සංඛ්‍යා	கலப்பு எண
Multiple	ගුණාකාර	மடங்கு
Multiplication	ගුණ කිරීම	பெருக்கல்
Negative integers	ඍණ නිඛිල	மறை நிறைவெண்
Number line	සංඛ්‍යා රේඛාව	எண்கோடு
Numerator	ලවය	தொகுதி
Obtuse angle	මහා කෝණය	பின்வளைகோணம்
Parallel lines	සමාන්තර සරල රේඛා	சமாந்தர நேர்கோடுகள்
Perpendicular distance	ලම්බ දුර	செங்குத்துத் தூரம்
Plus	ධන	நேர்
Positive integers	ධන නිඛිල	நேர் நிறைவெண்
Prime factors	ප්‍රථමක සාධක	முதன்மைக் காரணிகள
Proper fraction	තත්‍ය භාගය (නියම භාගය)	முறைமைப் பின்னம்
Protractor	කෝණමානය	பாகைமானி
Reflex angle	පරාවර්ත කෝණය	நிலைசார் எண்ணக்கரு
Right angle	ඍජු කෝණය	விரிகோணம்சிறியது
Set	කුලකය	தொடை
Set square	විහිත චතුරස්‍රය	மூலை மட்டம்
Side	බාහුව	புயம் (பக்கம்)
Static concept	ස්ථිතික සංකල්පය	இயக்கசார் எண்ணக்கரு
Straight edge	සරල දාරය	நேர்விளிம்பு (வரைகோல்)
Substitution	ආදේශය	பிரதியீடு
Subtraction	අඩු කිරීම	கழித்தல
Symmetrical plane figures	සමමිතික තල රූප	சமச்சீரான தளவுரு
Symmetry	සමමිතිය	சமச்சீர்
Unknowns	අඥාත	தெரியாக் கணியம
Unlike terms	විජාතීය පද	நிகரா உறுப்புக்கள்
Venn diagram	වෙන් රූප සටහන	வென் வரிப்படம
Vertex	ශීර්ෂය	உச்சி
Whole numbers	පූර්ණ සංඛ්‍යා	முழுஎண்கள்

Lesson Sequence

Content	Number of Periods	Competency levels
First Term		
1. Bilateral Symmetry	05	25.1
2. Sets	05	30.1
3. Mathematical Operations on Whole Numbers	04	1.1
4. Factors and Multiples	11	1.3, 1.4
5. Indices	06	6.1
6. Time	05	12.1
7. Parallel Straight Lines	03	27.1
8. Directed Numbers	06	1.2
9. Angles	07	21.1, 21.2
	52	
Second Term		
10. Fractions	10	3.1
11. Decimals	05	3.2
12. Algebraic Expressions	06	14.1, 14.2
13. Mass	06	9.1
14. Rectilinear Plane Figures	06	23.1, 23.2
15. Equations and Formulae	08	17.1, 19.1
16. Length	08	7.1, 7.2
17. Area	06	8.1
18. Circles	04	24.1
19. Volume	05	10.1
20. Liquid Measurements	04	11.1
	68	
Third Term		
21. Ratios	05	4.1
22. Percentages	05	5.1
23. Cartesian Plane	05	20.1
24. Construction of Plane Figures	05	27.2
25. Solids	05	22.1, 22.2
26. Data Representation and Interpretation	08	28.1, 29.1
27. Scale Diagrams	06	13.1
28. Tessellation	05	26.1
29. Likelihood of an Event	06	31.1, 31.2
	50	
Total	170	