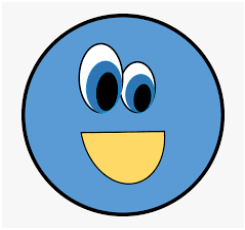

TO LEARN...

Chords of a circle

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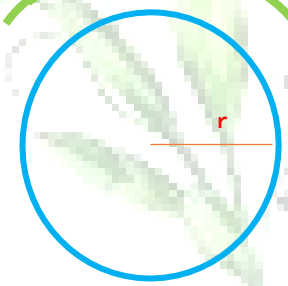
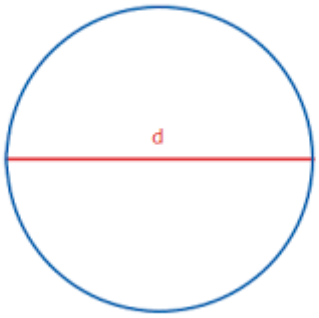
Let's begin,

Radius (r)

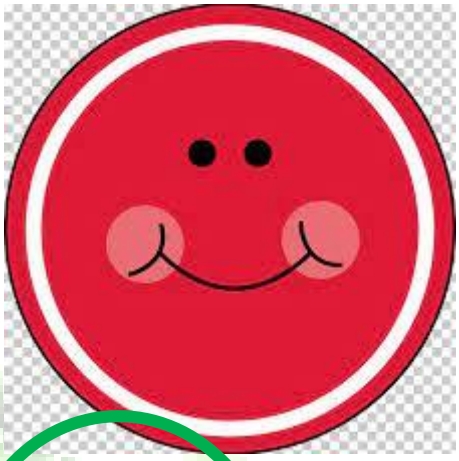


A straight line segment drawn from the centre of a circle to a point on the circle is called as the radius of the circle.

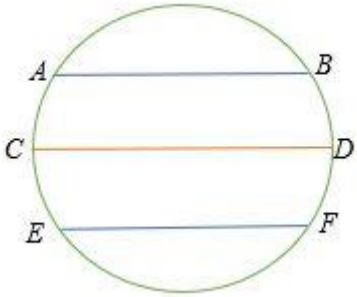
Diameter (d)



A straight line segment joining two points on the circle and passes through the centre of the circle is known as the diameter of the circle.



Chord

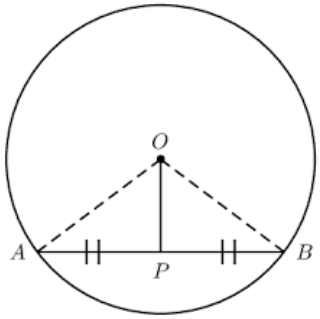


A straight line segment joining two points on the circle is defined as the chord of a circle.

- **The largest chord of a circle is its diameter.**

Theorem

The straight line joining the centre of the circle to the midpoint of a chord is perpendicular to the chord.



OP ⊥ ***AB***

The Formal Proof Of The Theorem

Data : O is the centre of the circle.

P is the midpoint of the chord AB.

To prove that: OP is perpendicular to AB.

Construction: Join OA and OB.

Proof : In the triangles OAP and OBP,

OA = OB (Radii of the circle)

AP = PB (P is the midpoint of AB)

OP is a common side

∴ OAP △ ≡ OBP △ (SSS)

$\therefore O\hat{P}A = O\hat{P}B$

But,

$O\hat{P}A + O\hat{P}B = 180^0$

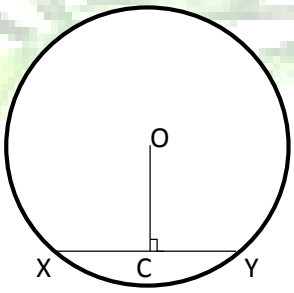
$\therefore 2O\hat{P}A = 180^0$

$O\hat{P}A = 90^0$

∴ OP is perpendicular to AB.

Converse of the theorem

The perpendicular drawn from the centre of a circle to a chord bisects the chord.



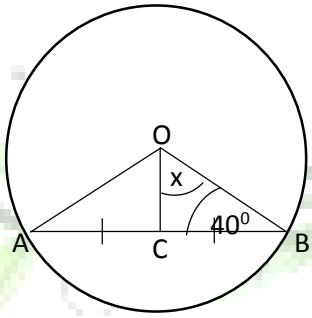
O is the centre of the circle and XY is a chord.

The perpendicular drawn from O to XY is OC.

Since OC ⊥ XY, C is the midpoint of XY. (XC = CY)

EXAMPLES:

1. According to the information given in the figure, find the value of x.



According to the theorem "The straight line joining the centre of the circle to the midpoint of a chord is perpendicular to the chord", OC ⊥ AB. ∴ OCB is a right triangle.

$O\hat{C}B = 90^0$

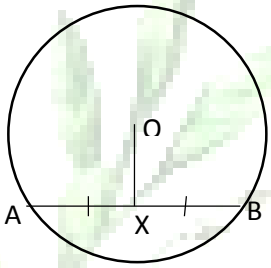
$90^0+40^0+x = 180^0$

$130^0+x = 180^0$

$130^0 - 130^0+x = 180^0-130^0$

$x = 50^0$

2. AB is a chord of a circle with centre O and radius 10cm. X is the midpoint of AB. If AB=12cm, find the length of OC.



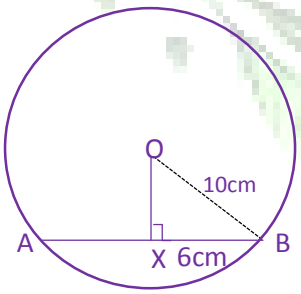
Since X is the midpoint of AB,

AX = XB.

AB = 12cm

∴ AX = XB = 6cm

According to the theorem "The straight line joining the center of the circle to the midpoint of a chord is perpendicular to the chord", OX ⊥ AB.



Using the Pythagorean relation,

$OB^2 = OX^2 + XB^2$

$10^2 = OX^2 + 6^2$

$100 = OX^2 + 36$

$100-36 = OX^2$

$64 = OX^2$

$\sqrt{64} = OX$

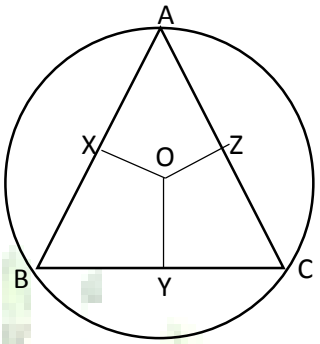
$8cm = OX$

$OX = 8cm$

3. ABC is an equilateral triangle. The points A, B and C are located on the circle with centre O. The perpendiculars drawn from O to the sides of the triangle AB, BC and AC are OX, OY and OZ respectively. If AZ = 5cm,

   I. Find the length of AC

   II. Find the perimeter of ABC Δ

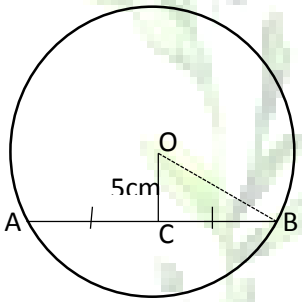


According to the converse of the theorem "The perpendicular drawn from the centre of a circle to a chord bisects the chord", OZ ⊥ AC and Z is the midpoint of the circle.

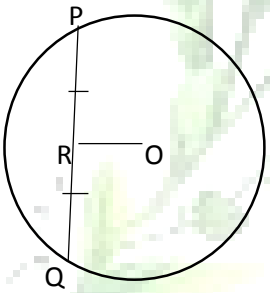
1) **AZ = ZC**
   AC = AZ + ZC
   = 5cm + 5cm
   = 10cm
2) **ABC is an equilateral triangle,**
   ∴ AB = BC = AC = 10cm
   Perimeter = 10cm × 3
   = 30cm

EXERCISE:

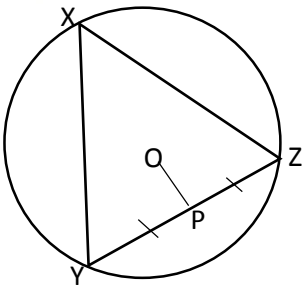
1) According to the information given in the figure,
   - I. Find the value of $O\hat{C}B$
   - II. If AB = 24cm, find the radius of the circle



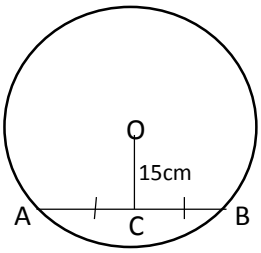
2) O is the centre of the circle with radius 10cm. If PQ = 16cm, according to the information given in the figure, find the length of OR.



3) XYZ is an equilateral triangle. The points X, Y and Z are located on the circle with centre O, OP=9cm and radius 15cm. According to the information given in the figure, find the perimeter of the XYZ triangle.



4) O is the centre of the circle with radius 25cm. According to the information in the figure, find the length of AB chord.



5) O is the centre of the circle. Based on the information given in the figure, find the value of x.