Sabaragamuwa Provincial Education Department – Weekly School

Mathematics

Grade: 10

Week: 12

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The Least Common Multiple of Algebraic Expressions

First, let us find what are factors.

As an example we take 6..

6 can be written as , $6 = 2 \times 3$, therefore 2 and 3 are factors of 6.

1. **LCM of some numbers**

   LCM of some numbers is the smallest number divisible by all that numbers.

   Let us find the LCM of 8, 12 and 24 by writing as a product of prime factors.

   $$8 = 2 \times 2 \times 2 = 2^3$$
   $$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$$
   $$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$$

   - 2 and 3 are the distinct prime factors of above numbers
   - To find LCM, we have to get the product of powers with the greatest indices.
   - The greatest power of $2 = 2^3$
     The greatest power of $3 = 3^1$
   - LCM $= 2^3 \times 3^1 = 8 \times 3 = 24$

2. **The LCM of some expressions with algebraic terms.**

   Let us find the LCM of $4x^2, 6xy \ and \ 8y$

   Now let us write the numerical terms of each term as a product of prime factors and, the algebraic terms as a product of distinct factors.

   $$4x^2 = 2 \times 2 \times x \times x = 2^2 \times x^2$$
   $$6xy = 2 \times 3 \times x \times y = 2^1 \times 3^1 \times x^1 \times y^1$$
   $$8y = 2 \times 2 \times 2 \times y = 2^3 \times y^1$$

   The distinct factors of above algebraic expressions are $2, 3, x \ and \ y$

   The greatest power of $2 = 2^3$

   The greatest power of $3 = 3^1$

   The greatest power of $x = x^2$

   The greatest power of $y = y^1$

Now let us get the LCM by taking the product of factors with the greatest powers.

LCM $= 2^3 \times 3^1 \times x^2 \times y^1 = 8 \times 3 \times x^2 \times y^1 = 24x^2y$

Let us consider the LCM of some algebraic expressions

i. $(y-x)^2, 2(x-y)^2, 4(x-y)$

$(y-x)^2 \quad = (x-y)^2$
$2(x-y)^2 = 2 \times (x-y)^2$
$4(x-y) = 2^2 \times (x-y)$
LCM $= 2^2 \times (x-y)^2 = 4(x-y)^2$

> *Remember*
>
> $(b-a)^2 \quad = (a-b)^2$

ii. $(2-x), \ x(x-2), \ 3x-6$

$(2-x) \quad = (-1) \times (x-2)$
$x(x-2) = x \times (x-2)$
$3x-6 \quad = 3 \times (x-2)$
LCM $= (-1) \times 3 \times x \times (x-2) = -3x(x-2)$

> *Remember*
>
> $(b-a) \quad = (-1)(a-b)$

iii. $2(x^2-y^2), 4(x+y), x(x-y)$

$2(x^2-y^2) = 2 \times (x-y) \times (x+y)$
$4(x+y) = 2^2 \times (x+y)$
$x(x-y) = x \times (x-y)$

> *Remember*
>
> $(a^2-b^2) \quad = (a-b)(a+b)$

LCM $= 2^2 \times x \times (x-y) \times (x+y) = 4x(x-y)(x+y)$

iv. $x^2-x-2, \ x^2-4, x^3-2x^2$

$x^2-x-2 \ = (x-2)(x+1)$
$x^2-4 \quad\ \ = (x-2)(x+2)$
$x^3-2x^2 \ \ = x^2(x-2)$

LCM $= x^2 \times (x-2) \times (x+2) \times (x+1) = x^2(x-1)(x+2)(x+1)$

Exercise 12.1, 12.2 and 12.3 of mathematics text book